tpwSpiral7feb04

adapted for spirals on Mixsig chip,
AMI 1.5 micron
28 sep 05

computes inductance and Q of spiral inductors
tweaked to fit measured data

$t := 0.0001\ \text{cm}$ $s := 0.0002\ \text{cm}$ $w := 0.0012\ \text{cm}$ $l_1 := 0.0190\ \text{cm}$ $N := 5$ (inputs)

t is metal thickness in cm
s is spacing between turns
w is width of conductor
l1 is length of 1st segment in cm
N=number of turns

$Rsh := 0.03\ \frac{\text{ohm}}{\text{sq}}$ $\rho si := .02\ \text{ohm}\cdot\text{cm}$

$\varepsilon r := 3.9$ $\varepsilon o := \frac{10^{-9}}{36\cdot\pi}$ $\varepsilon o = 8.8419\times 10^{-12}$

$hsi := 0.03\ \text{cm}$ $hsio := 0.00025\ \text{cm}$

$Cmetal := \varepsilon r\cdot\frac{\varepsilon o\cdot 10^{-12}}{hsio\cdot 10^{-2}}$ $Cmetal = 13.79343\times 10^{-18}$

Rsh is sheet resistivity of metal in ohm/square is metal thickness in cm
εr x εo is dielectric constant of insulator (typically SiO2)

h_{si} is thickness of silicon substrate (chip thickness)

h_{sio} is thickness of silicon oxide insulating layer between the metal and substrate

ρ_{si} is bulk resistivity of substrate in ohm-cm

N=number of turns

Cmetal = capacitance per square micron

$$k := 4 \cdot N$$

$$r := 2 .. (2 \cdot N)$$

$$l_2 := l_1 \qquad l_3 := l_1$$

$$l_{2 \cdot r} := l_2 - (r - 1) \cdot (w + s)$$

$$l_{2 \cdot r - 1} := l_1 - (r - 2) \cdot (w + s)$$

$$l_1 := l_1 + 0.5 \cdot (w + s)$$

$$length := \sum_{y=1}^{k} l_y$$

$l_1 = 0.0197$

$l_2 = 0.019$

$l_3 = 0.019 \qquad l_4 = 0.0176 \quad cm$

$length = 0.2673 \quad cm$

$$y := 1 .. k$$

$$L_y := 0.002 \cdot l_y \cdot \left[\ln\left[2 \cdot \frac{l_y}{(w + t)} \right] + 0.50049 + \left(\frac{w + t}{3 \cdot l_y} \right) \right]$$

$l =$

	0
0	0
1	0.0197
2	0.019
3	0.019
4	0.0176
5	0.0176
6	0.0162
7	0.0162
8	0.0148
9	0.0148
10	0.0134
11	0.0134
12	0.012
13	0.012

$$L_0 := \sum_{y=1}^{k} 10^3 \cdot L_y$$

$$L_0 = 1.9263 \quad \text{nH}$$

$$\text{GMD1} := \left| \begin{array}{l} \text{for } j \in 1..k \\ \quad \text{for } n \in 1..N \\ \quad\quad t_{j,(j+4\cdot n)} \leftarrow e^{\ln[n\cdot(s+w)] - \left[1 + \frac{1}{12\cdot\left[\frac{n\cdot(s+w)}{w}\right]^2} + \frac{1}{60\cdot\left[\frac{n\cdot(s+w)}{w}\right]^4} + \frac{1}{168\cdot\left[\frac{n\cdot(s+w)}{w}\right]^6}\right]} \text{ if } (j + 4\cdot n) \le k \\ \quad\quad 0 \quad \text{otherwise} \\ t \end{array} \right.$$

$$\text{M1j} := \left| \begin{array}{l} \text{sum1} \leftarrow 0 \\ \text{for } j \in 1..k \\ \quad \text{for } n \in 1..N \\ \quad\quad \text{sum1} \leftarrow \text{sum1} + 2\cdot l_j \cdot \left[\left[\ln\left[\frac{l_j}{\text{GMD1}_{j,(j+4\cdot n)}} + \left[1 + \left[\frac{l_j}{\text{GMD1}_{j,(j+4\cdot n)}}\right]^2\right]^{0.5}\right] - \left[1 + \left[\frac{\text{GMD1}_{j,(j+4\cdot n)}}{l_j}\right]^2\right]^{0.5}\right] + \frac{\text{GMD1}_{j,(j+4\cdot n)}}{l_j}\right] \text{ if } (\\ \quad\quad 0 \quad \text{otherwise} \\ \text{sum1} \end{array} \right.$$

$$
\begin{aligned}
&M1m := \left|\begin{array}{l}
sum1m \leftarrow 0 \\
\text{for } j \in 1..k \\
\quad \text{for } n \in 1..N \\
\quad\quad \left|\begin{array}{l} sum1m \leftarrow sum1m + 2 \cdot l_{j+4\cdot n} \cdot \left[\left[\left[\ln\left[\frac{l_{j+4\cdot n}}{GMD1_{j,(j+4\cdot n)}} + \left[1 + \left[\frac{l_{j+4\cdot n}}{GMD1_{j,(j+4\cdot n)}}\right]^2\right]^{0.5}\right] - \left[1 + \left[\frac{GMD1_{j,(j+4\cdot n)}}{l_{j+4\cdot n}}\right]^2\right]^{0.5}\right] + \frac{GMD1_{j,(j+} \\ 0 \quad otherwise \end{array}\right. \\
sum1m
\end{array}\right. \\
&M1p := \left|\begin{array}{l}
sum1p \leftarrow 0 \\
\text{for } j \in 1..k \\
\quad \text{for } n \in 1..N \\
\quad\quad \left|\begin{array}{l} sum1p \leftarrow sum1p + 2 \cdot \left(l_j - l_{j+4\cdot n}\right) \cdot \left[\left[\left[\ln\left[\frac{l_j - l_{j+4\cdot n}}{GMD1_{j,(j+4\cdot n)}} + \left[1 + \left[\frac{l_j - l_{j+4\cdot n}}{GMD1_{j,(j+4\cdot n)}}\right]^2\right]^{0.5}\right] - \left[1 + \left[\frac{GMD1_{j,(j+4\cdot n)}}{l_j - l_{j+4\cdot n}}\right]^2\right]^{0.5}\right] + \frac{GMD1_{j,} \\ 0 \quad otherwise \end{array}\right. \\
sum1p
\end{array}\right.
\end{aligned}
$$

$$M1 := M1j + M1m - M1p$$

$$M1 = 4.7356 \quad \text{nH}$$

$$GMD2 := \left| \begin{array}{l} \text{for } j \in 1..k \\ \quad \text{for } n \in 1..N \\ \quad\quad \left| \begin{array}{l} u_{j,(j+4\cdot n-2)} \leftarrow e^{\ln\left[l_{j+1}-n\cdot(s+w)\right]-\left[1+\frac{1}{12\cdot\left[\frac{l_{j+1}-n\cdot(s+w)}{w}\right]^2}+\frac{1}{60\cdot\left[\frac{l_{j+1}-n\cdot(w+s)}{w}\right]^4}+\frac{1}{168\cdot\left[\frac{l_{j+1}-n\cdot(s+w)}{w}\right]^6}\right]} \quad \text{if } (j+4\cdot n-2) \le k \\ 0 \quad \text{otherwise} \end{array} \right. \\ u \end{array} \right.$$

$$M2j := \left| \begin{array}{l} sum2 \leftarrow 0 \\ \text{for } j \in 1..k \\ \quad \text{for } n \in 1..N \\ \quad\quad \left| \begin{array}{l} sum2 \leftarrow sum2 + 2\cdot l_j\cdot\left[\left[\ln\left[\frac{l_j}{GMD2_{j,(j+4\cdot n-2)}}+\left[1+\left[\frac{l_j}{GMD2_{j,(j+4\cdot n-2)}}\right]^2\right]^{0.5}\right]-\left[1+\left[\frac{GMD2_{j,(j+4\cdot n-2)}}{l_j}\right]^2\right]^{0.5}\right]+\frac{GMD2_{j,(j+4\cdot n-}}{l_j} \\ 0 \quad \text{otherwise} \end{array} \right. \\ sum2 \end{array} \right.$$

$$
\begin{aligned}
M2m := \;& sum2m \leftarrow 0 \\
& \text{for } j \in 1..k \\
& \quad \text{for } n \in 1..N \\
& \quad\quad sum2m \leftarrow sum2m + 2 \cdot l_{j+4\cdot n-2} \cdot \left[\left[\left[\ln\left[\frac{l_{j+4\cdot n-2}}{GMD2_{j,(j+4\cdot n-2)}} + \left[1 + \left[\frac{l_{j+4\cdot n-2}}{GMD2_{j,(j+4\cdot n-2)}} \right]^2 \right]^{0.5} \right] - \left[1 + \left[\frac{GMD2_{j,(j+4\cdot n-2)}}{l_{j+4\cdot n-2}} \right]^2 \right]^{0.5} \right] + \frac{GM}{} \right. \\
& \quad\quad 0 \quad \text{otherwise} \\
& sum2m
\end{aligned}
$$

$$
\begin{aligned}
M2p := \;& sum2p \leftarrow 0 \\
& \text{for } j \in 1..k \\
& \quad \text{for } n \in 1..N \\
& \quad\quad sum2p \leftarrow sum2p + 2 \cdot \left(l_j - l_{j+4\cdot n-2} \right) \cdot \left[\left[\left[\ln\left[\frac{l_j - l_{j+4\cdot n-2}}{2 \cdot GMD2_{j,(j+4\cdot n-2)}} + \left[1 + \left[\frac{l_j - l_{j+4\cdot n-2}}{2 \cdot GMD2_{j,(j+4\cdot n-2)}} \right]^2 \right]^{0.5} \right] - \left[1 + \left[\frac{2 \cdot GMD2_{j,(j+4\cdot n-2)}}{l_j - l_{j+4\cdot n-2}} \right]^2 \right]^{(} \right. \right. \\
& \quad\quad 0 \quad \text{otherwise} \\
& sum2p
\end{aligned}
$$

$$M2 := M2j + M2m - M2p$$

$$M2 = 2.8417 \quad nH$$

$$Ltt := L_0 + M1 - M2$$

$$Ltt = 3.8202 \quad nH$$

$$Rsub_y := \frac{\rho si \cdot l_y}{w \cdot hsi}$$

$$Rp := \sum_{y=1}^{k} Rsub_y$$

$$Rs := \frac{length}{w} \cdot Rsh \qquad length = 0.2673$$

$$Rs = 6.6825 \quad ohms$$

$$Rp = 148.5 \quad ohms$$

$$Cp := \varepsilon o \cdot \varepsilon r \cdot \frac{w \cdot length}{hsi}$$

$$Cp = 3.687 \times 10^{-13} \quad F$$

$$Ls := Ltt \cdot 10^{-9}$$

$$f := 0.1 \cdot 10^{9}, 0.2 \cdot 10^{9} .. 5 \cdot 10^{9}$$

$$Rcp(f) := \frac{1}{j \cdot Cp \cdot 2 \cdot \pi \cdot f} \qquad Rls(f) := j \cdot Ls \cdot 2 \cdot \pi \cdot f$$

$$Z(f) := \frac{(Rp + Rcp(f)) \cdot (Rs + Rls(f))}{Rp + Rcp(f) + Rs + Rls(f)}$$

$$Q(f) := \frac{Im(Z(f))}{Re(Z(f))}$$

$$1 \qquad \left[1 - Rs^{2} \cdot \left(\frac{Cp}{Ls} \right) \right]^{\frac{1}{2}}$$

$$\omega r := \frac{1}{\sqrt{Ls \cdot Cp}} \cdot \left| \frac{}{1 - Rp^2 \cdot \left(\frac{Cp}{Ls}\right)} \right|$$

$$\omega r = 2.503i \times 10^{10}$$

$$\omega q := \frac{1}{\sqrt{Ls \cdot Cp}} \cdot \left[\frac{Rs}{2 \cdot Rp} \cdot \left[\left(1 + \frac{4}{3} \cdot \frac{Rp}{Rs} \right)^{\frac{1}{2}} - 1 \right] \right]^{\frac{1}{2}}$$

$$\omega q = 8.5109 \times 10^{9}$$

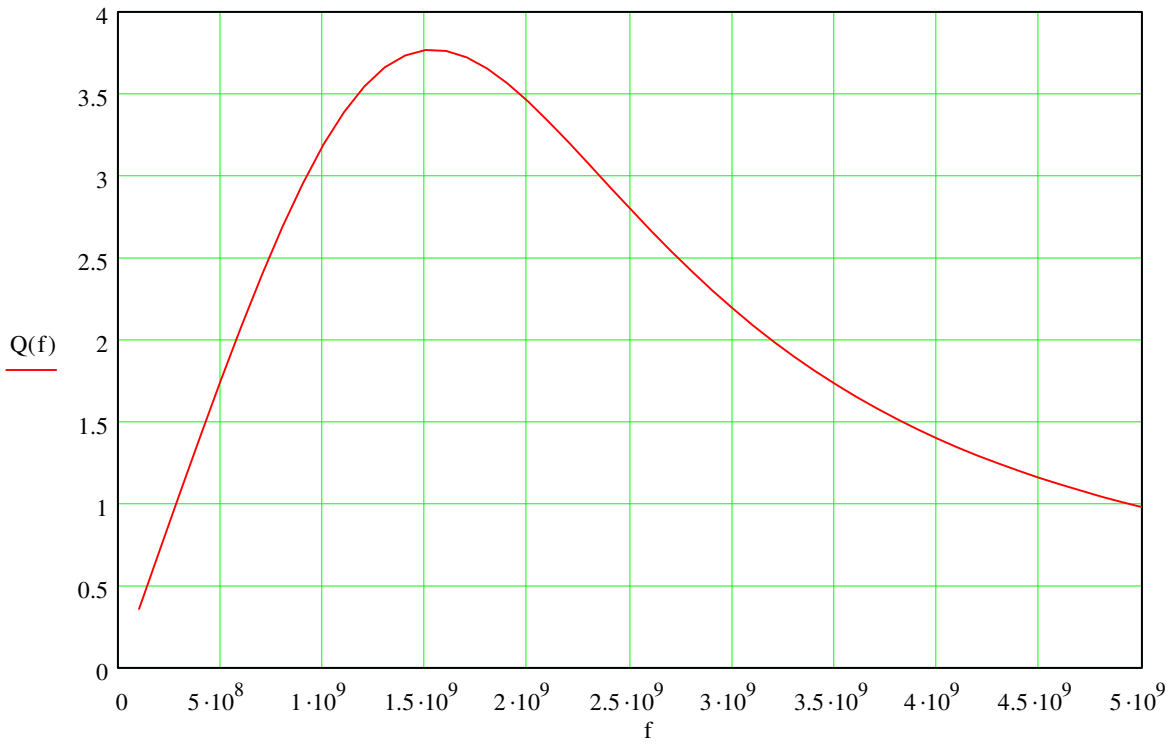
$$Q1(f) := \frac{Rs}{2 \cdot \pi \cdot f \cdot Ls} \cdot \left[1 - \left(\frac{f}{\omega r} \right)^2 \right]$$

$$fr := \frac{\omega r}{2 \cdot \pi}$$

$Rs = 6.6825$ $Rp = 148.5$

$Ls = 3.8202 \times 10^{-9}$ $Cp = 3.687 \times 10^{-13}$

$\omega r = 2.503i \times 10^{10}$ $fr = 3.9837i \times 10^{9}$ $\frac{1}{2 \cdot \pi \cdot \sqrt{Ls \cdot Cp}} = 4.2407 \times 10^{9}$



$j + 4 \cdot n) \leq k$

$$\frac{4 \cdot n)}{} \Bigg| \quad \text{if} \quad (j + 4 \cdot n) \leq k$$

$$\frac{(j+4 \cdot n)}{+4 \cdot n} \Bigg| \quad \text{if} \quad (j + 4 \cdot n) \leq k$$

$$\frac{\text{-2)}}{\text{}}\Bigg] \text{ if } (j + 4 \cdot n - 2) \leq k$$

$$\frac{D2_{j,(j+4\cdot n-2)}}{l_{j+4\cdot n-2}}\Bigg] \text{ if } (j+4\cdot n-2)\le k$$

$$).5\Bigg] + \frac{2\cdot GMD2_{j,(j+4\cdot n-2)}}{l_j - l_{j+4\cdot n-2}}\Bigg] \text{ if } (j+4\cdot n-2)\le k$$