Simulation of Simple Delta-Sigma Modulator for Driving Class C RF Amplifiers

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Abstract—**Delta-sigma modulators offer the potential for high linearity in a wide variety of radio frequency applications. In addition, the binary pulse output of the modulator motivates application as a driver for class C and class D amplifiers. Although class D amplifiers offer potential for higher power efficiency, class C amplifiers offer nearly the same performance with potential for less complexity. In addition, most radio signals of interest in these amplifier applications are bandpass. Therefore, the design and simulation of bandpass delta-sigma modulators are considered for application in driving class C amplifiers.**

I. Introduction

Delta-sigma modulators (DSM) have been well known for several decades, and have been applied to an increasingly wide range of applications. With high order loop filters and high over-sampling ratios, delta-sigma modulators have been used to create analog to digital converters (ADC) with more than 16 bits of resolution [1][2]. Because of the linearity and precision of single-bit feedback DSM architectures, the DSM has become a prime candidate for applications such as pulse width modulators and amplifier drivers [3][4]. Therefore, the present paper considers design characteristics of bandpass DSM systems for application in driving class C amplifiers.

Previous investigators have considered using DSM to drive class D RF amplifiers [4]. Such class D applications naturally lend themselves to a DSM architecture where the typical push-pull on/off type of class D amplifier output is well matched to a binary DSM output. In addition, researchers have considered DSM approaches to drive class C amplifiers, showing some promise for generating highly linear CDMA signals [3]. Although class D amplifier theoretical efficiency is 100%, class C amplifiers can theoretically reach 90% efficiency. Class C amplifiers are also typically simple single-supply single-transistor designs. Therefore, the present paper considers the design of DSM for radio frequency (RF) class C amplifiers, because of their competitive efficiency and low complexity.

A bandpass DSM is best suited as a driver for RF class C amplifiers, since the RF signals in transmitters are commonly bandpass in nature. In contrast to lowpass delta-sigma modulators that shape noise away from dc, a bandpass delta sigma modulators shape noise away from the band of interest [5]. This also allows the DSM to be designed to effectively shape and reduce noise near the frequency of operation.

In the next section, the design of lowpass first order DSM systems is first reviewed. Noise shaping characteristics of the DSM are also discussed. In the subsequent section, the lowpass first order DSM is modified to illustrate the design and performance of a simple bandpass DSM for use in driving RF amplifiers.

II. Lowpass first order delta-sigma modulator

The block diagram of a simple delta-sigma modulator is shown in Fig. 1. An input signal x[n] is subtracted from the feedback digital-to-analog converter (DAC) output, forming the input to the filter H(z). The output of filter H(z) is quantized to form output y[n]. Not shown in the figure are the additive quantization noise e[n] due to the quantizer, and any final decimation filter following y[n].

The delta-sigma modulator output Y(z) can be described by computing the closed-loop signal transfer function STF(z) of Fig. 1 and adding quantization noise shaped through a noise transfer function NTF(z). Taking the z-transform, X(z) is the input signal, E(z) is the quantization error, Y(z) is the output, and the system output can be shown to be:

$$Y(z) = \frac{H(z)}{1+H(z)}X(z) + \frac{1}{1+H(z)}E(z) \qquad (1)$$

or

$$Y(z) = STF(z)X(z) + NTF(z)E(z)\ . \qquad (2)$$

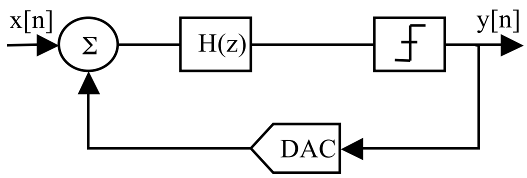

Figure 1. Simple delta-sigma modulator block diagram

For a simple lowpass sigma-delta modulator example, H(z) can be a delayed integrator described as

$$H(z) = \frac{z^{-1}}{1 - z^{-1}} \quad . \tag{3}$$

The modulator output now can be reduced to [6]:

$$Y(z) = z^{-1}X(z) + \left(1 - z^{-1}\right)E(z), \tag{4}$$

where E(z) is the error generated by the quantization process. Equation (4) shows that the output of the DSM equals a delayed version of the input, plus quantization error shaped by the effective highpass filtering of the frequency response of the term $(1\text{-}z^{-1})$.

Another approach to implement the first order delta-sigma modulator incorporates delay in the DAC feedback. In this case, the output Y(z) becomes:

$$Y(z) = \frac{H(z)}{1 + z^{-1}H(z)}X(z) + \frac{1}{1 + z^{-1}H(z)}E(z). \tag{5}$$

In addition, a non-delayed integrator H(z) can be employed with:

$$H(z) = \frac{1}{1 - z^{-1}} \quad . \tag{6}$$

Incorporating the delayed DAC from (5) and the non-delayed integrator from (6), the output Y(z) in (4) now becomes:

$$Y(z) = X(z) + \left(1 - z^{-1}\right)E(z) \quad . \tag{7}$$

The main difference between the two approaches in (7) and (4) is that the signal component of the output of the modulator is delayed in (4). However, from (4) and (7), it can be seen that noise transfer function NTF(z) is the same in both approaches.

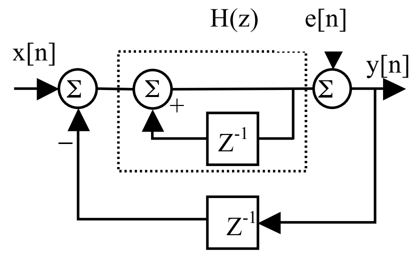

Figure 2. Simple delta-sigma modulator block diagram. H[z] is non delay integrator [7]

The complete block diagram of the lowpass first order delta sigma modulator for (7) is shown in Fig. 2. The loop inside the dashed box is the non-delayed integrator in (6) with z-transform H(z)=1/(1-z^{-1}). In the figure, x[n] and y[n] are again the input and output signals, and the quantization error e[n] is now explicitly shown. Thus, the theoretical output Y(z) for Fig. 2 is given by (7).

To illustrate operation of the DSM, the system of Fig. 2 was simulated for an input consisting of two sinusoids. Fig. 3 demonstrates that the reconstructed output signal tracks the input signal (after the output signal y[n] is post-processed in a decimator not shown in Fig. 2). The binary output y[n] is presented in Fig. 4, and shows a string of low outputs when the reconstructed output is low, and a string of high outputs when the reconstructed output signal is high.

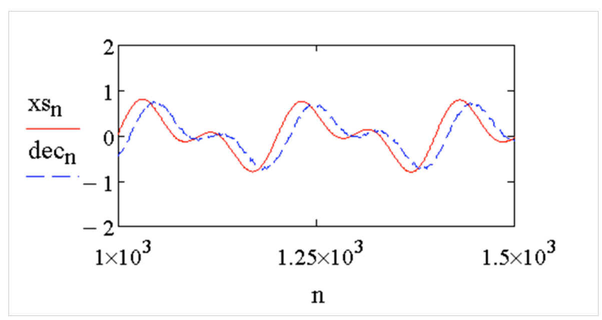

Figure 3. Reconstructed two-tone output signal (dash blue) in comparison to two-tone input (solid red)

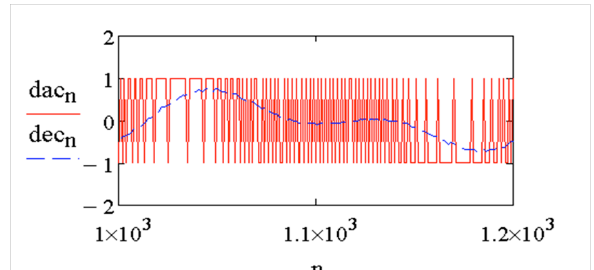

Figure 4. Reconstructed signal (dash blue) and corresponding binary DAC output.

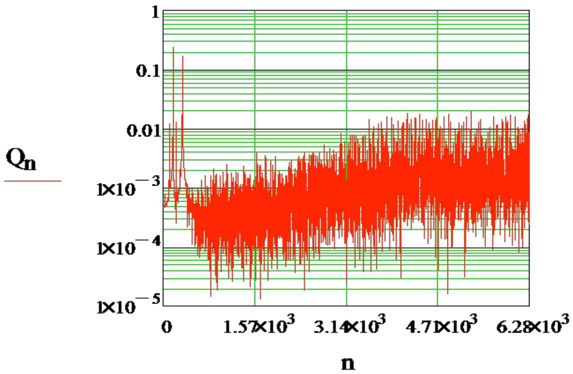

Figure 5. Output spectrum obtained directly from the y[n] output without filtering or decimation. Frequency axis corresponds to ω from 0 to π rad/sample.

III. BANDPASS DELTA SIGMA MODULATION

A simple method for creating a bandpass DSM transfer function is by replacing z by $-z^2$ in the lowpass DSM. This converts the low pass STF(z) into a bandpass signal transfer function, and converts the highpass NTF(z) into a bandstop noise transfer function. The transformation can be viewed as moving the single lowpass pole at z=1 to a pair of bandpass poles on the unit circle at z=±j. Thus, the transformed H(z) behaves as a resonator with poles at z=±j. By using the resonator for H(z), the modulator effectively shapes noise away from the signal passband of interest, pushing the noise toward dc and ω=π rad/sample.

The transfer function of a second order band pass resonator H(z) shown in Fig. 7 can be described as:

$$H(z) = \frac{1}{1+z^{-2}} \tag{8}$$

Following the same lines as for the lowpass DSM, the closed loop transfer function of Fig. 7 can be described as:

$$Y(z) = X(z) + \left(1+z^{-2}\right)E(z) \tag{9}$$

As can be seen from (9), the signal transfer function STF(z) is not changed, while the noise transfer function

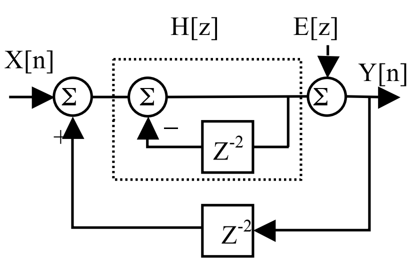

Figure 7. Simple band pass delta-sigma modulator block diagram[7]

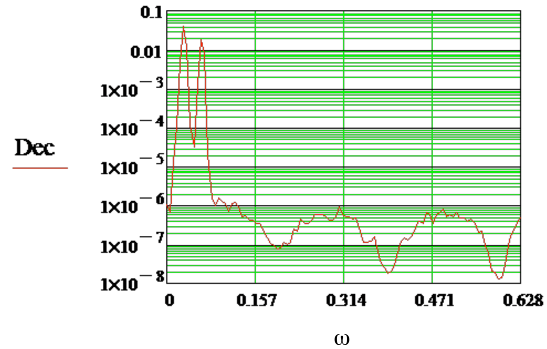

Figure 6. Decimator output spectrum

NTF(z) has a second order zero at ω=π/2 rad/sample. This second order zero suppresses the noise at the passband. As before, Fig. 8 shows the input signal compared to the reconstructed signal. Fig. 9 shows the binary output y[n] along with the reconstructed signal.

For the bandpass DSM, Fig. 10 shows the first N/2 points of a discrete Fourier transform of y[n]. The sinusoid comprising the input signal is visible as a sharp peak near ω=π/2 rad/sample. The characteristic bandstop behavior of the noise transfer function NTF(z) is also seen in the noise floor of Fig. 10. Figure 11 shows the Fourier transform of the reconstructed signal after decimation. The frequency axis in Fig. 11 is discrete frequency ω from 0 to π rad/s.

The simple bandpass example in (9) illustrates the capability of a DSM in driving a class C or class D amplifier. From Fig. 10, the binary modulator output y[n] is seen to pass

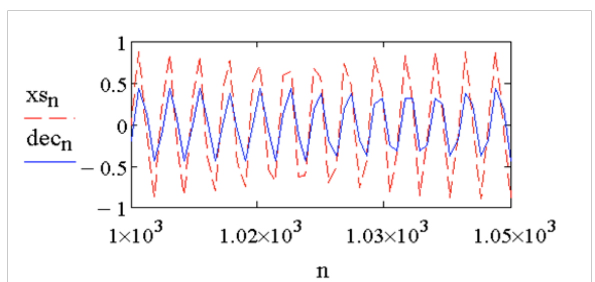

Figure 8. Reconstruct output of band pass delta sigma modulator. There are delays between input and output, however, the shape of the output signals still replica that of the input.

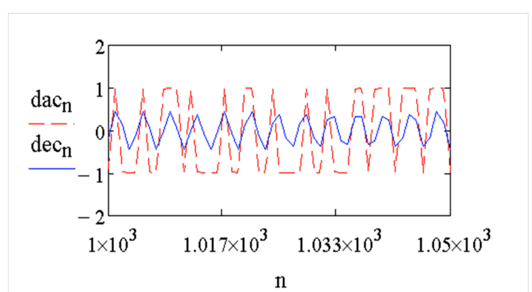

Figure 9. Reconstruct DAC output signal correspond to the DAC output.

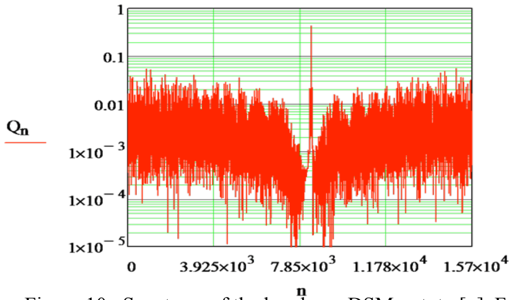

Figure 10. Spectrum of the bandpass DSM outut y[n]. Frequency axis corresponds to ω from 0 to π rad/sample.

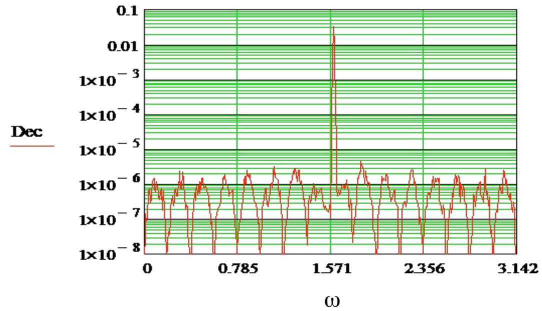

Figure 11. Spectrum of the reconstructed band pass modulator after decimation filtering.

the desired bandpass RF signal, as required. In addition, the quantization noise in Fig. 10 is seen to be suppressed near the RF frequency by the noise transfer function NTF(z). In addition, the binary output streams shown in Figs. 4 and 9 are well suited to driving on/off amplifier stages, such as class D and class C amplifiers [3],[4].

For a band pass DSM, the out of band quantization noise will affect nearby channels, especially in wide band application. Improvement may be obtained by adding pseudo white noise to the modulator[8]. Reducing noise correlation can improve further SNR [9].

IV. FUTURE WORK

The foregoing examples illustrate the main features of DSM. In particular, the bandpass DSM has the desired signal transfer function and noise transfer function for bandpass RF signals. In addition, the binary output is well-suited to driving RF amplifier stages where the transistors operate as on/off switches. Because of the simplicity of class C stages, future plans are focusing on using the binary output of the bandpass DSM to drive class C RF amplifiers. Current efforts are focusing on duty cycle issues to maintain class C operation.

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