A Digital Non-Foster VHF Radio Approach for Enabling Low-Power Internet of Things

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***Abstract*—A digital non-Foster radio approach is proposed to mitigate Wheeler-Chu limits of electrically-small antennas, with significant potential to significantly reduce energy consumption in the VHF (very high frequency) band, where radio propagation losses below 200 MHz are 100 times less than losses above 2 GHz. Operation at lower frequency could greatly extend lifetimes of small low-power Internet-of-Things devices such as battery-powered sensors operating primarily as transmitters. Unfortunately, physical size constraints and the Wheeler-Chu limit have greatly hindered utilization of VHF bands for mobile devices, where even a 200 MHz half-wave dipole is an unwieldy 0.75 m. However, recent advances in non-Foster impedance matching methods have overcome these limits. In addition, recent digital non-Foster methods are shown to closely resemble digital radio architectures, suggesting that these newer digital non-Foster methods can be readily adopted in new digital radio designs. Therefore, a novel digital non-Foster radio architecture is proposed, where digital non-Foster methods enable small devices in energy-efficient VHF bands while overcoming Wheeler-Chu antenna-size limits.**

I. INTRODUCTION

Significantly lower propagation losses at VHF (very high frequency) offer potential to improve energy efficiency by more than a factor of 100, since radio propagation losses below 200 MHz are 100 times less than losses above 2 GHz [1], [2], and since electrically-small antennas can have gain within a few dB of a half-wave dipole [3]. However, the fundamental physics of small mobile antennas have prevented use of VHF bands, such that a 2010 FCC report [4] lists antenna size constraint as a primary limitation in efficient VHF band utilization [5]. In particular, the Wheeler-Chu limit for electrically-small antennas shows antenna bandwidth decreasing in proportion to the inverse of the cube of antenna size [3], [6]–[8].

However, investigators have recently demonstrated that the Wheeler-Chu limit *can be overcome by employing non-Foster methods* to significantly improve the bandwidth of electrically-small antennas [9]–[15]. These revolutionary advances have been achieved through antenna impedance matching with non-Foster circuits, such as negative capacitors and negative inductors [16]–[18]. Early experiments demonstrated enhancement greater than 10 dB between two antennas over bands as wide as 30-200 MHz [13], with similar results in [14] and [15].

More recently, *digital non-Foster methods* have been developed, offering the potential for software-reconfigurable, wideband, adaptive, enhancement of small antennas over a broad range of frequencies [19]–[23]. Importantly, the architecture of digital non-Foster circuits lends itself to straightforward implementation in modern digital radio architectures such as SDRs (software-defined radios), where the needed ADC (analog-to-digital converter) could be a component of the digital-radio receiver, and the needed DAC (digital-to-analog converter) could be a component of the digital-radio transmitter [24]–[28]. Furthermore, the digital tunability of a digital non-Foster approach offers potential for adaptive stabilization against instabilities induced by antenna impedance variations caused by nearby objects [28], [29]. Such stability issues are a significant design concern in analog non-Foster approaches [30], but can be mitigated by adaptive methods and inherent upper frequency bounds of the Nyquist limit in digital non-Foster circuits [23], [28].

Therefore, a *digital non-Foster radio architecture* for Internet-of-Things is proposed to overcome Wheeler-Chu limits of small antennas and capitalize on lower VHF propagation loss. The proposed digital non-Foster radio architecture takes advantage of similarity between modern digital radio architectures and recently-introduced digital non-Foster circuits. In essence, a digital signal-processing path is proposed to be added between the digital receiver and digital transmitter to generate desired non-Foster impedances suitable tor impedance matching of electrically small antennas [28]. The resulting non-Foster impedance of the radio can then overcome Wheeler-Chu limits [28]. Furthermore, the digital non-Foster approach [20]–[23], [31]–[33] offers the potential to address stability issues often encountered in analog non-Foster approaches [12], [16], [17], [30], and dynamic adaptive tuning and stabilization to compensate for variation of mobile antenna impedance caused by nearby objects and antenna movement [28]. Finally, *lower-frequency signal processing and hardware of a digital VHF radio* should benefit from innate performance advantages relative to digital radio designs above 1 GHz.

The following section summarizes motivation and background on VHF propagation, electrically-small antennas, digital non-Foster circuits, and related advantages and features. Then, section III discusses the proposed digital non-Foster radio architecture, associated digital non-Foster impedance, and related advantages and features. A full transceiver is considered for completeness, because of the wide-ranging requirements of radios such as commercial-broadcast receivers (receive-only), transmit-only sensors, frequency division du-

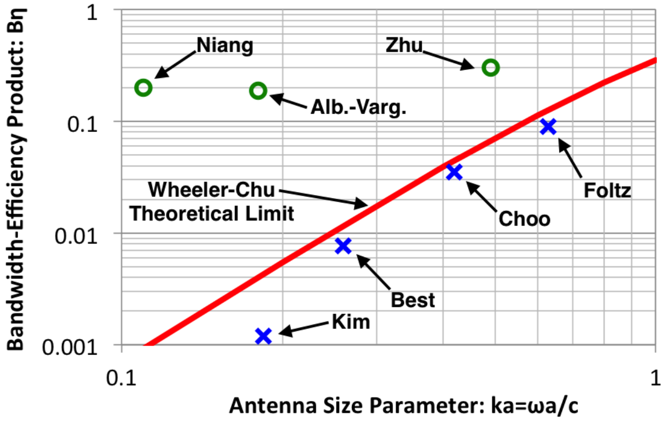

Fig. 1. Plot of normalized antenna bandwidth-efficiency parameter $B\eta$ as a function of antenna size parameter $ka = \omega a/c = 2\pi a/\lambda$ for conventional and non-Foster matching networks of electrically-small antennas. Solid red curve is Wheeler-Chu limit, blue crosses ($\times$) are measured passively-matched antennas, green circles ($\circ$) are measured non-Foster-matched antennas (≈*hundredfold improvement near ka=0.1)*.

plex radios, and time-division duplex radios [34]. *Nevertheless, small antennas and power-efficiency are common requirements in mobile devices, and are the primary focus of the proposed VHF digital non-Foster radio architecture.*

II. BACKGROUND AND MOTIVATION

The proposed digital non-Foster radio architecture draws upon VHF propagation characteristics and several recent developments in non-Foster devices and antennas. This section provides a summary of advantages of VHF propagation, analog non-Foster antenna enhancement beyond Wheeler-Chu limits, similarities between digital non-Foster circuits and digital radio architectures, and fundamentals of digital non-Foster circuits.

A. Decreased Propagation Losses at VHF

One key motivation for developing radio architectures capable of digital non-Foster impedance matching of electrically-small VHF antennas is the potential for reducing transmitter power by a factor of 100. According to Friis and Hata path loss models, radio propagation losses increase by a factor of 100 for every factor of 10 increase in frequency [1], [2]. For example, a factor of 7.5 frequency reduction could potentially increase a 1-week transmitter battery life to as much as 195 weeks, in a transmit-only device. This follows from the Hata large-urban model propagation loss L in dB:

$$\begin{aligned} L = \; & 69.55 + 26.16 \, \log_{10}(f) - 13.82 \, \log_{10}(h_B) \\ & -C_H + [44.9 - 6.55 \, \log_{10}(h_B)] \, \log_{10}(d) \;, \quad (1) \end{aligned}$$

where $C_H = 3.2[\log_{10}(11.75 \, h_M)]^2 - 4.97$, frequency f is in MHz, d is distance in kilometers, and h_B and h_M are base-station and mobile antenna heights in meters. Then, for fixed antenna heights, the difference in loss for two frequencies f_1 and f_2 is $\Delta L = 26.16 \, \log_{10}(f_1/f_2)$. So, the radio propagation loss at 1.5 GHz is 22.9 dB greater than the loss at 200 MHz, or a factor of 195 greater loss. Thus, potential for hundredfold transmitter power reduction exists even if antenna efficiency becomes as low as 51%, since electrically-small antennas can have gain within a few dB of a half-wave dipole [3], [35].

B. Non-Foster Enhancement Beyond Wheeler-Chu Limit

A second important motivation for developing digital non-Foster radio architectures for small VHF antennas is the potential for extraordinary improvement in antenna bandwidth. A serious impediment associated with electrically-small antennas is the fundamental Wheeler-Chu limit where bandwidth severely decreases with decreasing antenna size [6]–[8], [36], with bandwidth-efficiency product [3]:

$$B\eta = \frac{1}{\sqrt{2}/(k\,a) + \sqrt{2}/(k\,a)^3} \qquad (2)$$

for a linear polarization antenna where VSWR=2, B is fractional bandwidth, η is antenna efficiency, a is radius of a sphere that would enclose the antenna, $ka = \omega a/c = 2\pi a/\lambda$, and signal free-space wavelength is λ. To illustrate the severity of this issue in VHF at 200 MHz where wavelength $\lambda = 1.5$ m, a 10 cm dipole with $ka = 0.21$ would have a bandwidth limit of only $B\eta \approx 1.25$ MHz.

Fig. 1 shows the tremendous amount of bandwidth improvement that can be achieved using non-Foster impedance matching for electrically-small antennas. In Fig. 1, normalized antenna bandwidth-efficiency parameter $B\eta$ is plotted as a function of antenna size parameter ka for conventional and non-Foster antenna impedance matching networks. The solid red curve in Fig. 1 is the Wheeler-Chu theoretical limit using (2) and parameters noted in [3]. The blue crosses are measured data points for representative passively-matched electrically-small antennas with parameters taken from [3] for antenna designs in Kim [37], Best [38], Choo [39], and Foltz [40]. The green circles are measured data points for non-Foster-matched electrically-small antennas from Niang [9], Albarracín-Vargas [10], and Zhu [11], where $\eta = 0.5$ if not given (in line with theory in [41] and with Wheeler [35] reporting $\eta = 0.5$ for $ka = 0.12$). *Note that the non-Foster 125 MHz antenna of Niang [9] outperforms the Wheeler-Chu limit by a factor of more than 100, near* $ka = 0.1$. Thus, non-Foster methods can overcome the fundamental bandwidth limitations of electrically-small antennas at VHF.

C. Similarity of Non-Foster and Digital Radio Architectures

Another motivation for a digital non-Foster radio architecture for impedance matching of electrically-small VHF antennas is the similarity of the digital non-Foster circuit in Fig. 2(a) to the architecture of an “ideal” digital radio illustrated in Fig. 2(b), where the combiner in Fig. 2(b) usually includes a circulator, duplexer, etc., in place of the “ideal” direct connection shown. Nevertheless, it is apparent in Fig. 2 that digital radios *may already include* much of the ADC, DAC, samplers, and computational hardware that may be needed to implement digital non-Foster impedance matching. Thus, it may be relatively straightforward to add non-Foster functionality to some digital radio designs *with minimal added hardware or signal processing*.

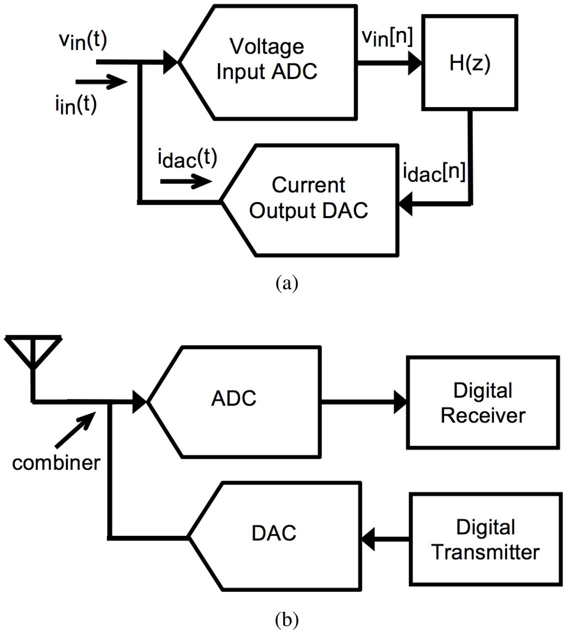

Fig. 2. Comparison of digital non-Foster and digital radio architectures. (a) Block diagram of digital non-Foster circuit [20], where input voltage $v_{in}(t)$ is digitized by the ADC, then filtered by $H(z)$ to form input current $i_{in}(t)$ from the DAC. (b) Block diagram of "ideal" digital radio.

D. Digital Non-Foster Circuit Theory

Before leaving Fig. 2, the theory of the digital non-Foster circuit of Fig. 2(a) is briefly reviewed [20]. To generate a non-Foster port impedance at $v_{in}(t)$ of Fig. 2(a), the digitized input voltage $v_{in}[n] = v_{in}(nT)$ out of the ADC with period T is processed by a discrete-time filter with z-transform $H(z)$. The digital filter output is then $i_{dac}[n] = v_{in}[n] * h[n]$, setting the input current $i_{in}(t) = i_{dac}(t)$ through the DAC [20]. The port impedance $Z_{in}(s) = V_{in}(s)/I_{in}(s)$ is then [20]

$$Z_{in}(s) \approx \left.\frac{V_{in}(z)}{I_{in}(z)}\right|_{z=e^{sT}} = \left.\frac{sT}{H(z)(1-z^{-1})}\right|_{z=e^{sT}} , \qquad (3)$$

assuming a ZOH (zero-order hold) in the DAC giving rise to the term $(1-z^{-1})/(sT)$ in the expression, $v_{in}(t)$ is properly sampled without any aliasing, and frequencies are less than $0.5/T$ Hz. For example, the circuit can be made to look like a capacitor with *positive or negative capacitance* C by setting $H(z) = C(1-z^{-1})/T$, or made to look like a *positive or negative inductance* L, when $H(z) = T/[L(1-z^{-1})]$. Details for latency effects, stability, noise, negative RLC impedances, and Thévenin forms are in references [20]–[23], [31]–[33], [42].

III. PROPOSED RADIO ARCHITECTURE

This section first presents the proposed digital non-Foster radio architecture. Then, the input impedance of the receiver and output impedance of the transmitter are derived, since the present work is focused on non-Foster impedance matching of electrically small antennas. In addition, related topics of Nyquist stability advantages, adaptive impedance estimation, generalized "digital impedance radios," and antenna-mismatch preselection filtering are noted. Although space does not permit detailed discussion of these many related design issues, a number of related papers discuss stability issues [23], [43], noise and noise shaping [32], adaptive digital antenna impedance matching [28], digital impedance estimation [28], and latency issues [32], [43].

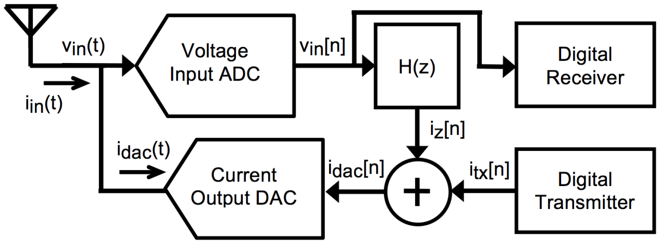

Fig. 3. Proposed digital non-Foster radio architecture, where digital signal processing H(z) determines radio input impedance and output impedance.

A. Digital Non-Foster Radio Architecture

Drawing upon the foregoing discussion of digital non-Foster methods, the proposed "ideal" digital non-Foster radio architecture for electrically-small antennas is shown in Fig. 3. As in Fig. 1(b), an "ideal" scenario is considered for simplicity, where transmitter and receiver signals are combined by direct connection instead of through a circulator, duplexer, etc. A full transceiver is illustrated in Fig. 3 for completeness, even though devices such as a transmit-only sensor may not need a full receiver, and receive-only devices such as commercial-broadcast receivers would not require a transmitter or duplexer. Furthermore, tradeoffs vary widely in different radio designs, where alternatives such as time-division duplex and frequency-division duplex greatly affect radio architecture [34]. Nevertheless, common requirements for battery-powered mobile devices are electrically-small antennas and low power consumption, and motivate the proposed digital non-Foster radio architecture. Finally, the Thévenin forms given in [33] could also be used in Fig. 3.

B. Receiver Input Impedance

To derive the receiver input impedance in Fig. 3, let the ADC input impedance and DAC output impedance be infinite, so that $i_{in}(t) = i_{dac}(t)$. Then, the input voltage at the antenna $v_{in}(t)$ is first digitized by the ADC to form $v_{in}[n] = v_{in}(nT)$, at a sample rate of $1/T$ Hz. Signal $v_{in}[n]$ is both processed by the digital filter with z-transform $H(z)$, and is later processed by the digital receiver stages to demodulate incoming signals from the antenna. The digital filter output $i_z[n] = v_{in}[n] * h[n]$ with $I_z(z) = V_{in}(z)H(z)$ is used to establish the impedance seen by the antenna (typically a non-Foster impedance for present purposes). The transmitter current $i_{tx}[n]$ is added to $i_z[n]$ to form the total current $i_{dac}[n]$, where the DAC output current is $i_{dac}(nT) = i_{in}(nT) = i_{dac}[n] = i_{tx}[n] + i_z[n]$. When $i_{tx}[n] = 0$, the transmitter is disabled, and $I_{dac}(z) = V_{in}(z)H(z)$. The receiver input impedance $Z_{rx}(s)$ is then

$$Z_{rx}(s) = \frac{V_{in}(s)}{I_{in}(s)} \approx \left.\frac{sT}{H(z)(1-z^{-1})}\right|_{z=e^{sT}} , \qquad (4)$$

for $v_{in}(t)$ sampled without any aliasing, a ZOH in the DAC, and for frequencies below 0.5/T Hz. Note that the result in (4) is the same as in (3) for the digital non-Foster circuit of Fig. 2(a).

C. *Transmitter Output impedance*

Next, the transmitter source impedance for Fig. 3 is analyzed by taking the ratio of the open-circuit voltage when the antenna is disconnected, divided by the current when the antenna is replaced by a short circuit. The short-circuit current follows from $I_{dac}(z) = V_{in}(z)H(z)+I_{tx}(z)$ with $V_{in}(z) = 0$, resulting in short-circuit current $I_{txsc}(z) = -I_{tx}(z)$, since the source current direction should be out of the port. For the open-circuited antenna voltage, now let R_e be the equivalent Norton source resistance of the DAC in parallel with the ADC, so that $I_{dac}(z) = -V_{in}(z)/R_e = V_{in}(z)H(z)+I_{tx}(z)$, and rearranging, $V_{in}(z) = -I_{tx}(z)R_e/[1 + H(z)R_e]$. For an ideal ADC and DAC, $R_e \to \infty$, and the open-circuit voltage becomes $V_{inoc}(z) = -I_{tx}(z)/H(z)$. Thus, the transmitter source impedance is $Z_{tx}(s) = V_{inoc}(s)/I_{txsc}(s)$, or

$$Z_{tx}(s) \approx \frac{-I_{tx}(z)/H(z)}{-I_{tx}(z)} \frac{sT}{1-z^{-1}}\bigg|_{z=e^{sT}} = Z_{rx}(s)\ . \qquad (5)$$

Thus, the transmitter and receiver port impedance are equal, and the impedance is established by the signal processing $H(z)$ of the digital non-Foster radio in Fig. 3.

D. *Nyquist Stability Advantage*

Stability is a common issue that must be addressed in the design of analog and digital systems containing non-Foster circuits [18], [23], [30], [43]. In analog non-Foster circuits, oscillations often occur at high frequencies, well above the intended band of non-Foster circuit operation. One added advantage of digital non-Foster circuits is that any oscillation must be constrained to frequencies less than half the clock rate of the ADC and DAC, because of the Nyquist limit [44]. This constraint on possible oscillation frequency range is an advantage of digital implementations of non-Foster systems.

E. *Adaptive Antenna Impedance Estimation*

The impedance of a mobile antenna may be affected by nearby objects or manufacturing variation, and such variation in impedance could lead to instability when interfacing with a non-Foster load [29]. Therefore, adaptive antenna impedance estimation methods [28] and tunable digital non-Foster devices [45] are being explored. In particular, antenna impedance estimation is incorporated into the signal processing of digital non-Foster systems such as those in Fig. 2(a) and Fig. 3, where ARMA models and/or pulse responses are used to estimate the antenna impedance and adjust antenna non-Foster compensation parameters [28]. The digital non-Foster approach is well-suited to adaptive methods, since the non-Foster behavior can be changed by variation of $H(z)$, embedded software, clock frequencies, etc. Further details are in [28], [45]

F. *General Digital Impedance Radios and Hybrids*

Although the foregoing discussion has focused on digital non-Foster radio design with impedances such as negative capacitance, is also possible to design $H(z)$ to present "normal" impedances, such as positive capacitance, positive inductance, and as a positive RLC resonator. This is clearly possible, since equations (3), (4), and (5) are not restricted to non-Foster impedances, and can more generally synthesize impedances such as positive capacitance and inductance. Also, it may be useful to construct hybrid impedance matching methods where added external devices, such as an external positive inductance, or a transformer, etc., may provide some intermediate impedance transformation between the antenna and digital non-Foster radio of Fig. 3.

G. *Antenna as Preselector Filter*

In some applications, it may be advantageous to use the mismatch of an electrically-small antenna to provide some degree of preselection filtering. For example, a conventional simple series inductor matching into an electrically-short monopole would result in a narrowband impedance match that could be employed as preselection filter. Thus, designing a digital impedance radio with a positive inductive impedance (as mentioned in the previous section) could provide some degree of preselection. Further, use of a more general RLC digital impedance as in [42] could also provide greater control of preselection bandwidth.

IV. SUMMARY

A novel digital non-Foster radio architecture is proposed for Internet-of-Things, where digital non-Foster methods enable small mobile devices in energy-efficient VHF bands while overcoming Wheeler-Chu limits of small antennas. Energy efficiency is achieved through utilization of VHF bands with less propagation loss. Small wideband VHF antennas are achieved by incorporating digital non-Foster methods into a digital radio architecture. Although much future work remains beyond the present limited scope, several earlier works begin to lay foundations on digital non-Foster stability issues [23], [43], noise shaping [32], antenna impedance estimation [28], latency issues [32], [43], and overall digital non-Foster methods [19], [20], [28], [46]. A related approach appears in [46], but with a priority date several weeks later than [19]. Similarly, emerging Floquet impedance matching methods may offer another approach for overcoming the antenna Wheeler-Chu limit [47].

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REFERENCES

[1] H. T. Friis, "A note on a simple transmission formula," *Proc. of the IRE*, vol. 34, no. 5, pp. 254–256, May 1946.

[2] M. Hata, "Empirical formula for propagation loss in land mobile radio services," *IEEE Trans. Veh. Technol.*, vol. 29, no. 3, pp. 317–325, Aug. 1980.

[3] D. F. Sievenpiper, D. C. Dawson, M. M. Jacob, T. Kanar, S. Kim, J. Long, and R. G. Quarfoth, “Experimental validation of performance limits and design guidelines for small antennas,” *IEEE Trans. Antennas Propag.*, vol. 60, no. 1, pp. 8–19, Jan. 2012.

[4] Federal Communications Commission (FCC), “Spectrum analysis: Options for broadcast spectrum OBI technical paper no. 3,” available on FCC website at www.fcc.gov/general/national-broadband-plan, Jun. 2010.

[5] Federal Communications Commission (FCC), “Connecting America: Nat. broadband plan,” available on FCC website at www.fcc.gov/general/national-broadband-plan, Mar. 2010.

[6] H. Wheeler, “Fundamental limitations of small antennas,” *Proc. of the IRE*, vol. 35, no. 12, pp. 1479–1484, Dec. 1947.

[7] L. J. Chu, “Physical limitations of omnidirectional antennas,” *Journal of Applied Physics*, vol. 19, no. 12, pp. 1163–1175, 1948.

[8] H. Wheeler, “The radiansphere around a small antenna,” *Proc. of the IRE*, vol. 47, no. 8, pp. 1325–1331, Aug. 1959.

[9] A. Niang, A. de Lustrac, G. P. Piau, S. Villers, and S. N. Burokur, “Wideband miniaturized antenna with external non-Foster circuit,” in *2016 IEEE Antennas and Propag. Soc. Int. Symp. (APSURSI)*, Jun. 2016, pp. 1729–1730.

[10] F. Albarracín-Vargas, D. Segovia-Vargas, and V. Gonzalez-Posadas, “Non-Foster matching network for a small, bladetype monopole in the VHF band,” in *2015 IEEE Antennas and Propag. Soc. Int. Symp. (APSURSI)*, Jul. 2015, pp. 1396–1397.

[11] N. Zhu and R. W. Ziolkowski, “Active metamaterial-inspired broad-bandwidth, efficient, electrically small antennas,” *IEEE Antennas Wireless Propag. Lett.*, vol. 10, pp. 1582–1585, 2011.

[12] S. E. Sussman-Fort and R. M. Rudish, “Non-Foster impedance matching of electrically-small antennas,” *IEEE Trans. Microw. Theory Techn.*, vol. 57, no. 8, pp. 2230–2241, Aug. 2009.

[13] C. White, J. Colburn, and R. Nagele, “A non-Foster VHF monopole antenna,” *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 584–587, 2012.

[14] Y. Shen and T. H. Chio, “Matching electrically small antenna using non-Foster circuit in the FM-band,” in *Antennas and Propagation (EuCAP), 2014 8th European Conf. on*, Apr. 2014, pp. 333–336.

[15] A. M. Elfrgani and R. G. Rojas, “Stability of non-Foster circuits for broadband impedance matching of electrically small antennas,” in *Radio and Wireless Symp. (RWS), 2015 IEEE*, Jan. 2015, pp. 50–52.

[16] R. M. Foster, “A reactance theorem,” *Bell Syst. Tech. J.*, vol. 3, pp. 259–267, 1924.

[17] J. Linvill, “Transistor negative-impedance converters,” *Proc. of the IRE*, vol. 41, no. 6, pp. 725–729, Jun. 1953.

[18] E. Ugarte-Munoz, S. Hrabar, D. Segovia-Vargas, and A. Kiricenko, “Stability of non-Foster reactive elements for use in active metamaterials and antennas,” *IEEE Trans. Antennas Propag.*, vol. 60, no. 7, pp. 3490–3494, Jul. 2012.

[19] T. P. Weldon, “Digital discrete-time non-Foster circuits and elements,” published on WIPO patentscope.wipo.int as WO2015164866 on Oct. 29, 2015, Int. Patent Application No. PCT/US2015/27789 filed Apr. 27, 2015.

[20] T. P. Weldon, J. M. C. Covington, K. Smith, and R. S. Adams, “Performance of digital discrete-time implementations of non-Foster circuit elements,” in *2015 IEEE Int. Symp. on Circuits and Systems (ISCAS)*, May 2015, pp. 2169–2172.

[21] T. P. Weldon, J. M. C. Covington, K. L. Smith, and R. S. Adams, “A two-port digital discrete-time non-Foster circuit designed for negative capacitance,” in *Advanced Electromagnetic Materials in Microwaves and Optics (METAMATERIALS), 2015 9th Int. Congr. on*, Sep. 2015, pp. 319–321.

[22] T. P. Weldon and R. S. Adams, “A digital discrete-time non-Foster approach to broadband fast-wave microstrip lines,” in *Advanced Electromagnetic Materials in Microwaves and Optics (METAMATERIALS), 2015 9th Int. Congr. on*, Sep. 2015, pp. 322–324.

[23] T. P. Weldon, J. M. C. Covington, K. Smith, and R. S. Adams, “Stability conditions for a digital discrete-time non-Foster circuit element,” in *2015 IEEE Antennas and Propag. Soc. Int. Symp. (APSURSI)*, Jul. 2015.

[24] J. Mitola, “The software radio architecture,” *IEEE Commun. Mag.*, vol. 33, no. 5, pp. 26–38, May 1995.

[25] P. Cruz, N. B. Carvalho, and K. A. Remley, “Designing and testing software-defined radios,” *IEEE Microw. Mag.*, vol. 11, no. 4, pp. 83–94, Jun. 2010.

[26] J. Tsutsumi, M. Seth, A. S. M. III, R. B. Staszewski, and G. Hueber, “Cost-efficient, high-volume transmission: Advanced transmission design and architecture of next generation RF modems and front-ends,” *IEEE Microw. Mag.*, vol. 16, no. 7, pp. 26–45, Aug. 2015.

[27] Y. Chen, S. Lu, H. S. Kim, D. Blaauw, R. G. Dreslinski, and T. Mudge, “A low power software-defined-radio baseband processor for the Internet of Things,” in *2016 IEEE Int. Symp. on High Performance Computer Architecture (HPCA)*, Mar. 2016, pp. 40–51.

[28] K. K. Steer, P. J. Kehoe, and T. P. Weldon, “Investigation of an adaptively-tuned digital non-Foster approach for impedance matching of electrically-small antennas,” in *IEEE SoutheastCon 2017*, Mar. 2017, pp. 1–5.

[29] K. R. Boyle, Y. Yuan, and L. P. Ligthart, “Analysis of mobile phone antenna impedance variations with user proximity,” *IEEE Trans. Antennas Propag.*, vol. 55, no. 2, pp. 364–372, Feb. 2007.

[30] S. Stearns, “Incorrect stability criteria for non-Foster circuits,” in *2012 IEEE Antennas and Prop. Soc. Int. Symp. (APSURSI)*, Jul. 2012, pp. 1–2.

[31] K. L. Smith, R. S. Adams, and T. P. Weldon, “Measurement of a fast-wave line using digital non-Foster circuits for software-adjustable delay,” in *2016 IEEE Antennas and Propag. Soc. Int. Symp. (APSURSI)*, Jun. 2016, pp. 193–194.

[32] K. K. Steer, P. J. Kehoe, and T. P. Weldon, “Effects of latency and quantization noise on digital discrete-time non-Foster circuits,” in *2016 IEEE Antennas and Propag. Soc. Int. Symp. (APSURSI)*, Jun. 2016, pp. 731–732.

[33] P. J. Kehoe, K. K. Steer, and T. P. Weldon, “Thevenin forms of digital discrete-time non-Foster RC and RL circuits,” in *2016 IEEE Antennas and Propag. Soc. Int. Symp. (APSURSI)*, Jun. 2016, pp. 191–192.

[34] B. Razavi, *RF Microelectronics*, 2nd ed., ser. Prentice Hall Comm. Eng. and Emerging Tech. Series. Upper Saddle River, NJ, USA: Prentice Hall Press, 2011.

[35] H. Wheeler, “Small antennas,” *IEEE Trans. Antennas Propag.*, vol. 23, no. 4, pp. 462–469, Jul. 1975.

[36] J. McLean, “A re-examination of the fundamental limits on the radiation Q of electrically small antennas,” *IEEE Trans. Antennas Propag.*, vol. 44, no. 5, pp. 672–676, May 1996.

[37] O. S. Kim, “Low-Q electrically small spherical magnetic dipole antennas,” *IEEE Trans. Antennas Propag.*, vol. 58, no. 7, pp. 2210–2217, Jul. 2010.

[38] S. R. Best, “Low Q electrically small linear and elliptical polarized spherical dipole antennas,” *IEEE Trans. Antennas Propag.*, vol. 53, no. 3, pp. 1047–1053, Mar. 2005.

[39] H. Choo, R. L. Rogers, and H. Ling, “Design of electrically small wire antennas using a Pareto genetic algorithm,” *IEEE Trans. Antennas Propag.*, vol. 53, no. 3, pp. 1038–1046, Mar. 2005.

[40] H. D. Foltz, J. S. McLean, and G. Crook, “Disk-loaded monopoles with parallel strip elements,” *IEEE Trans. Antennas Propag.*, vol. 46, no. 12, pp. 1894–1896, Dec. 1998.

[41] C. Pfeiffer, “Fundamental efficiency limits for small metallic antennas,” *IEEE Trans. Antennas Propag.*, vol. 65, no. 4, pp. 1642–1650, Apr. 2017.

[42] K. K. Steer, P. J. Kehoe, and T. P. Weldon, “Measurement of a digital non-Foster negative RLC circuit and digital positive RLC circuit,” in *IEEE SoutheastCon 2017*, Mar. 2017, pp. 1–5.

[43] P. J. Kehoe, K. K. Steer, and T. P. Weldon, “Stability analysis and measurement of RC and RL digital non-Foster circuits with latency,” in *IEEE SoutheastCon 2017*, Mar. 2017, pp. 1–4.

[44] C. Phillips and H. Nagle, *Digital Control System Analysis and Design*. Prentice-Hall, 1990.

[45] P. J. Kehoe, K. K. Steer, and T. P. Weldon, “Measurement of a clock-tuned digital non-Foster circuit for positive or negative digital capacitance,” in *IEEE SoutheastCon 2017*, Mar. 2017, pp. 1–5.

[46] F. C. Robey, T. M. Hancock, and G. B. Stahl, “Digital matching of a radio frequency antenna,” published on WIPO patentscope.wipo.int as WO2016028354 on Feb. 25, 2016, Int. Patent Application No. PCT/US2015/033742 filed Jun. 2, 2015.

[47] H. Li, A. Mekawy, and A. Alù, “Beyond Chu’s limit with Floquet impedance matching,” *Phys. Rev. Lett.*, vol. 123, p. 164102, Oct. 2019.