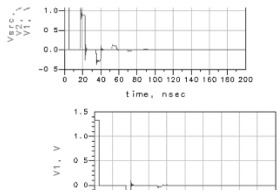
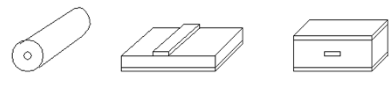
Waves and Reflection, Pulses

- In high speed systems, you may have observed multiple reflections of pulses on circuit boards



- Therefore, transmission lines, waves, reflections serve important role in RF/microwave systems
- Optics and reflections are a great analogy

Transmission Lines

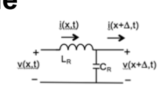


- Coaxial - Typically 50 or 75 ohms
- Twin-lead UHF - typ 300 ohms
- Microstrip
- Stripline
- Waveguide
- Coplanar waveguide
- Slot line

Waves, LC Transmission Line

Orfanidis 10.6

- Why transmission lines at high frequency:
  - Finite velocity, time delay, and ***waves!***
  - A ground wire may be an open circuit!



Lossless LC line relations:

$$\frac{\partial v(x,t)}{\partial x} = -L_R \frac{\partial i(x,t)}{\partial t} \Rightarrow \nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\frac{\partial i(x,t)}{\partial x} = -C_R \frac{\partial v(x,t)}{\partial t} \Rightarrow \nabla \times H = \varepsilon \frac{\partial E}{\partial t},$$

Note: k is wavenumber, or spatial frequency,and when purely real has units of radians/meter

$$v(x,t) = v_+(x,t) + v_-(x,t) = \mathrm{Re}\{V_+ e^{-jkx} e^{j\omega t} + V_- e^{jkx} e^{j\omega t}\}$$

$$v_+(x,t) = \mathrm{Re}\{V_0 e^{-jkx} e^{j\omega t}\} = \mathrm{Re}\{V_0 e^{-\gamma x} e^{j\omega t}\} = \mathrm{Re}\{V_0 e^{-(\alpha + j\beta)x} e^{j\omega t}\}$$

Phasor notation drops $e^{j\omega t}$: $E_0 e^{-\gamma z} e^{j\omega t}$ becomes $E_0 e^{-\gamma z}$

and $\nabla \times H = J + \varepsilon \partial E / \partial t \Rightarrow \nabla \times H = J + j\omega\varepsilon E$ *with* $E = E_0 e^{-\gamma z} e^{j\omega t}$

Finally propagation constant $\gamma = \alpha + j\beta$ and phase velovity v_p

$\gamma = j\omega / v_p = j\omega\sqrt{L_R C_R}$, so wavelength $\lambda = v_p / f = 2\pi / \beta$

and characteristic impedance $Z_0 = v_\pm(x,t) / i_\pm(x,t) = \sqrt{L / C}$

Beware of Literature

- It is common in engineering literature to solve circuit equations, maxwell equations, and transmission line equations using phasors of the form exp(jwt):

Phasor notation with $e^{j\omega t}$: $E_0 e^{-\gamma z} e^{j\omega t}$ becomes phasor $E_0 e^{-\gamma z}$

and $\nabla \times H = J + \varepsilon \partial E / \partial t \Rightarrow \nabla \times H = J + j\omega\varepsilon E$

- Beware of physics/optics literature (use of i vs j ***may*** forewarn) solves maxwell equations and transmission line equations using phasors of the form exp(-iwt):

Phasor notation with $e^{-i\omega t}$: $E_0 e^{ikz} e^{-i\omega t}$ becomes phasor $E_0 e^{ikz}$

and $\nabla \times H = J + \varepsilon \partial E / \partial t \Rightarrow \nabla \times H = J + -i\omega\varepsilon E$

whereas an engineering text may use $E_0 e^{-jkz} e^{j\omega t}$

Notably: Yariv of Cal Tech is In the engineering camp

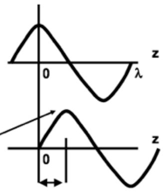
Traveling Wave

Orfanidis 3.5

- Phase velocity

Time t=0, θ=0, α=0
Re{V_p $e^{j(\omega t + \theta)}$ $e^{-\alpha z}$ $e^{-j\beta z}$}
= cos(ωt -βz)
= cos(-βz)

Time t=Δt, θ=0, α=0
Re{V_p $e^{j(\omega t + \theta)}$ $e^{-\alpha z}$ $e^{-j\beta z}$}
= cos(ωt -βz)
= cos(ωΔt -βz)



Peak is where 0=ωΔt -βΔz
So ωΔt = βΔz and Δz= ωΔt /β
Then velocity = Δz/Δt = ω/β

Group Velocity

Orfanidis 3.5

- Group velocity is velocity of energy travel

group velovity $v_g = d\omega / d\beta$ is velocity of energy packet

consider: $\cos(a) + \cos(b) = 2\cos\left((a-b)/2\right)\cos\left((a+b)/2\right)$

for $\cos(\omega_1 t - \beta_1 x) + \cos(\omega_2 t - \beta_2 x) = 2\cos\left((\Delta\omega t - \Delta\beta x)/2\right)\cos\left((\Sigma\omega t - \Sigma\beta x)/2\right)$

where $\beta = \omega / v_p$, $\Delta\omega = \omega_1 - \omega_2$, and $\Delta\beta = \beta_1 - \beta_2$

at $t = 0$, the peak of the envelope is at $x = 0$, at $t = \Delta t$, the peak moves to Δx where

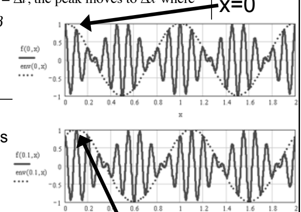
$\left(\Delta\omega\,\Delta t - \Delta\beta\,\Delta x\right)/2 = 0$ *so* $\Delta x = \Delta\omega\,\Delta t / \Delta\beta$

then, group velovity v_g is the envelope velocity

$$v_g = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta\beta \to 0} \frac{\Delta\omega}{\Delta\beta} = \frac{\partial\omega}{\partial\beta}$$

Envelope

At t=0, peak at x=0



At t=0.1, peak at x=0.1

For the lossles LC line

$v_p = \omega / \beta = 1 / \sqrt{L_R C_R}$,

so, $\omega = \beta / \sqrt{L_R C_R}$, then

$v_g = \frac{\partial\omega}{\partial\beta} = 1 / \sqrt{L_R C_R} = v_p$

Example:
$v_g = v_p$ =1 m/s
f_1 =11 Hz
f_2 =9 Hz

Note: vg does not always equal vp

Lossy Line

Orfanidis 10.6

- Include resistance R (ohm/m) in series with L (H/m)
- Include conductance G (S/m) in parallel with C (F/m)

Lossy RLGC line propagation constant and phase velocity v_p

$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$,

where $V_0 e^{-\gamma x} e^{j\omega t} = V_0 e^{-(\alpha + j\beta)x} e^{j\omega t} = V_0 e^{-\alpha x} e^{-j\beta x} e^{j\omega t}$

and then $\beta = \mathrm{Im}\{\gamma\} = \mathrm{Im}\{\sqrt{(R + j\omega L)(G + j\omega C)}\}$

so wavelength $\lambda = v_p / f = 2\pi / \beta$

and phase velocity $v_p = \omega / \beta$

and attenuation constant α is

$\alpha = \mathrm{Re}\{\gamma\} = \mathrm{Re}\{\sqrt{(R + j\omega L)(G + j\omega C)}\}$

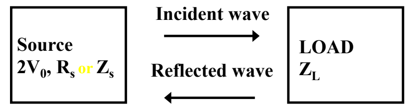
and characteristic impedance

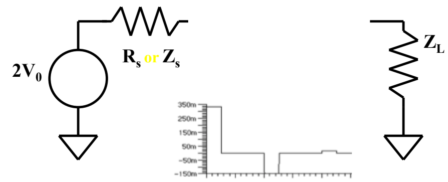
$Z_0 = \sqrt{(R + j\omega L)/(G + j\omega C)}$

Time DomainReflections

Orfanidis 10.15

See Ludwig Fig. 2-23, 2-24 and 2-34





Refletion Coefficient

- It is easiest to analyze wave phenomena using the reflection coefficient, gamma:

$\Gamma = \dfrac{Z_L - Z_S}{Z_L + Z_S}$ where Z_L is source and Z_S is load impedance

Note: approach here may differ from some textbooks

At input interface, incident wave $V_+ = V_s / 2$

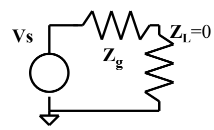
At input interface, reflected wave $V_- = \Gamma V_+$

At input interface, total voltage $V_T = V_+ + V_- = (1+\Gamma)V_s / 2 = \dfrac{Z_L V_s}{Z_L + Z_S}$

As a pulse makes multiple reflections, repeat the above process where the source impedance becomes the line impedance, and the load alternately becomes the generator and load

Simple Examples Without a Line

- Short circuit load



$\Gamma = \dfrac{Z_L - Z_S}{Z_L + Z_S} = \dfrac{0 - Z_g}{0 + Z_g} = -1$

incident wave $V_+ = V_s / 2$

reflected $V_- = \Gamma V_+ = -V_+ = -V_s / 2$

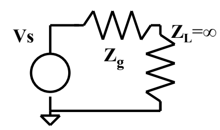
total voltage:

$V_T = V_+ + V_- = (1+\Gamma)V_s / 2 = 0$

As expected,

0 volts across the short circuit

- Open circuit load



$\Gamma = \dfrac{Z_L - Z_S}{Z_L + Z_S} = \dfrac{\infty - Z_g}{\infty + Z_g} = 1$

incident wave $V_+ = V_s / 2$

reflected $V_- = \Gamma V_+ = V_+ = -V_s / 2$

total voltage:

$V_T = V_+ + V_- = (1+\Gamma)V_s / 2 = V_s$

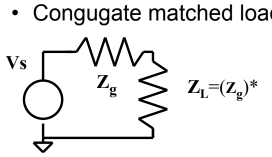
As expected,

all volts across the open circuit

© 2001-2015 T.Weldon

Simple Examples Without a Line

- Congugate matched load



$\Gamma = \dfrac{Z_L - Z_S}{Z_L + Z_S} = \dfrac{Z_g^* - Z_g}{Z_g^* + Z_g} = \dfrac{-2j\,\mathrm{Im}\{Z_g\}}{2\,\mathrm{Re}\{Z_g\}} = \dfrac{-j\,\mathrm{Im}\{Z_g\}}{\mathrm{Re}\{Z_g\}}$

incident wave $V_+ = V_s / 2$

reflected $V_- = \Gamma V_+ = -V_+ = -V_s / 2$

total voltage:

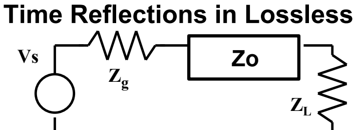
$V_T = V_+ + V_- = (1+\Gamma)V_s / 2 = \dfrac{\left(\mathrm{Re}\{Z_g\} - j\,\mathrm{Im}\{Z_g\}\right)V_s}{2\,\mathrm{Re}\{Z_g\}} = \dfrac{Z_g^* V_s}{Z_g^* + Z_g}$

As expected,

the conjugate match voltage equals that of the voltage divider comprised of Z_g^* and Z_g

Time Reflections in Lossless Line

Orfanidis 10.15 and See http://thomasweldon.com/tpw/papers/aps2013_extraction_6jan13f_subppt.pdf



incident wave $V_{+1} = V_s / 2$

$\Gamma_1 = \dfrac{Z_L - Z_S}{Z_L + Z_S} = \dfrac{Z_0 - Z_g}{Z_0 + Z_g}$

$\Gamma_1 V_+ = \dfrac{Z_0 - Z_g}{Z_0 + Z_g}\dfrac{V_s}{2}$

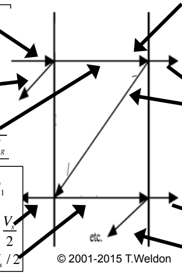
$V_{+1} = V_{T1} = (1+\Gamma_1)\dfrac{V_s}{2} = \dfrac{Z_0 V_s}{Z_0 + Z_g}$

incident $V_{+1} = (1+\Gamma_1)V_s / 2$

$\Gamma_2 = \dfrac{Z_L - Z_S}{Z_L + Z_S} = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$V_{T2} = (1+\Gamma_2)V_{+1} = (1+\Gamma_2)(1+\Gamma_1)\dfrac{V_s}{2}$

$V_{-2} = \Gamma_2 V_{+1} = \Gamma_2 (1+\Gamma_1)V_s / 2$



$\Gamma_3 = \dfrac{Z_L - Z_S}{Z_L + Z_S} = \dfrac{Z_g - Z_0}{Z_g + Z_0} = -\Gamma_1$

$V_{T3} = (1+\Gamma_3)V_{-2} = \Gamma_2(1-\Gamma_1^2)\dfrac{V_s}{2}$

$V_{+3} = \Gamma_3 V_{-2} = -\Gamma_1\Gamma_2(1+\Gamma_1)V_s / 2$

incident $V_{+3} = \Gamma_3\Gamma_2(1+\Gamma_1)V_s / 2$

$\Gamma_4 = \Gamma_2 = \dfrac{Z_L - Z_S}{Z_L + Z_S} = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$V_{T4} = -\Gamma_1\Gamma_2(1+\Gamma_2)(1+\Gamma_1)\dfrac{V_s}{2}$

$V_{-4} = \Gamma_4 V_{+3} = -\Gamma_1\Gamma_2^2(1+\Gamma_1)\dfrac{V_s}{2}$

© 2001-2015 T.Weldon

Short Pulse Simulation

R_S= 100, Z_0 = 50, R_L= 25
Transmission line delay = 100 ns
Open-circuit source voltage Vs=1 V

$$\Gamma_1 = \frac{Z_L - Z_S}{Z_L + Z_S} = \frac{Z_0 - R_s}{Z_0 + R_s} = \frac{50-100}{50+100} = -0.33$$

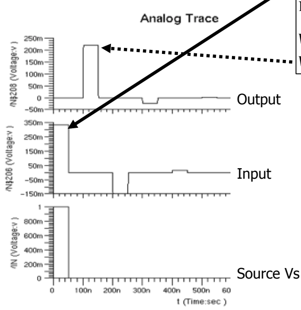
first reflection $\Gamma_1 V_+ = -0.33 V_s / 2 = -0.165$

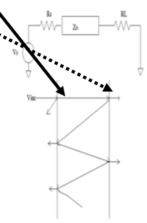
$$V_{+1} = V_{T1} = (1+\Gamma_1)V_s / 2 = (1-0.33)/2 = 0.33$$

$$\Gamma_2 = \frac{Z_L - Z_S}{Z_L + Z_S} = \frac{R_L - Z_0}{R_L + Z_0} = \frac{25-50}{25+50} = -0.33$$

$$V_{-2} = \Gamma_2 V_+ = -0.33 V_{+1} = -0.11$$

$$V_{T2} = (1+\Gamma_2)V_{+1} = (1+-0.33)(0.33) = 0.22$$





Decibels

- Decibels, or dB, are 10 $\log_{10}$ (power ratio)
  - 10 $\log_{10}$ (P / P_{ref})
- Always power ratio!
- Typically relative to some reference
  - 1V, 1W, milliwatt, μV
- Notation:dBx = dB relative to reference x
- dBm = dB relative to 1 milliwatt (Ludwig p. 491)
- Power in dBm units =
  - 10 $\log_{10}$ (power in milliwatts /1 milliwatt)
- Example: 0.0001 Watts =
  - 10 $\log_{10}$ (0.1 mW / 1 mW}) = -10 dBm

Decibels 2

- Power is proportional to voltage squared or current squared
- So translating a voltage ratio into power ratio requires square
- Factors of 2 in power=10dB, factors of 10 in power=10dB
- dB = 10 $\log_{10}$ (power ratio) =
  - 10 $\log_{10}$ [V^2 / $(V_{ref})^2$] =20 $\log_{10}$ (V / V_{ref})
- The 20 log comes from the square of the voltage
- Example: 10 V rms = 20 dBv
- What is 5 mW in dBm? In dBW?

- Project 1