

Z-TRANSFORM

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad \text{p. 14}$$

$$x[n] = \frac{1}{j2\pi} \oint X(z) z^{n-1} dz$$

- ① $X(z)$ MUST INCLUDE ROC (REGION OF CONVERGENCE)
- ② closed contour MUST BE IN ROC

Z-TRANSFORM IS ANALOGOUS TO LAPLACE TRANSFORM

Common Z-TRANSFORM PAIRS

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$$\textcircled{1} \delta[n] \xleftrightarrow{\mathcal{Z}} 1 \quad \text{ROC} = \text{everywhere}$$

$$\textcircled{2} u[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{1-z^{-1}} \quad \text{ROC } |z| > 1$$

$$\textcircled{3} a^n u[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{1-az^{-1}} \quad \text{ROC } |z| > |a|$$

$$\textcircled{4} -a^n u[-n-1] \xleftrightarrow{\mathcal{Z}} \frac{1}{1-az^{-1}} \quad \text{ROC } |z| < |a|$$

PROPERTIES

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① BIBO STABLE IF ROC INCLUDES UNIT CIRCLE

$$\textcircled{2} \mathcal{Z}\{x[n-n_0]\} = z^{-n_0} X(z)$$

$$\textcircled{3} \mathcal{Z}\{x[n]*h[n]\} = X(z)H(z), \text{ ROC} = \text{INTERSECTION}$$

$$\textcircled{4} \mathcal{Z}\{\alpha^n x[n]\} = X(z/\alpha), \text{ ROC} = |\alpha| \times \text{SCALE} \text{ ROC} \\ \text{ie } |z| > |\alpha| r_0$$

INVERSE Z-TRANSFORM METHODS

- ① CONTOUR INTEGRAL $x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$
- ② USE PARTIAL FRACTION EXPANSION, THEN USE LOOK-UP TABLE OF Z-TRANSFORMS
- ③ Power Series Expansion (LONG DIVISION)

① Example: $X(z) = \frac{z}{z-0.1}$ ROC: $|z| > 0.1 \Rightarrow$ RIGHT SIDED

$$\begin{array}{r}
 1 + 0.1z^{-1} + 0.01z^{-2} \dots \\
 z-0.1 \overline{) z} \\
 \underline{z-0.1} \\
 0.1 \\
 0.1 - 0.01z^{-1} \\
 \underline{0.01z^{-1}} \\
 \dots
 \end{array}
 \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{COEFFICIENT of } z^{-n} \text{ gives } x[n] \\ \text{so } x[0] = 1 \\ x[1] = 0.1 \\ x[2] = 0.01 \text{ etc} \end{array}$$

② IF $X(z)$ WAS LISTED IN POWERS OF z^{-1} , i.e. $X(z) = \frac{1}{1-0.1z^{-1}}$

$$\begin{array}{r}
 1 + 0.1z^{-1} + 0.01z^{-2} \dots \\
 1 - 0.1z^{-1} \overline{) 1} \\
 \underline{1 - 0.1z^{-1}} \\
 0.1z^{-1} \\
 0.1z^{-1} - 0.01z^{-2} \\
 \underline{0.01z^{-2}} \\
 \dots
 \end{array}
 \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{SAME ANSWER} \\ \text{AS BEFORE!} \end{array}$$

③ However, IF ROC WAS $|z| < 0.1 \Rightarrow$ LEFT SIDED

$$\begin{array}{r}
 -10z - 100z^2 - 1000z^3 \dots \\
 -0.1 + z \overline{) z} \\
 \underline{z - 10z^2} \\
 10z^2 \\
 10z^2 - 100z^3 \\
 \underline{100z^3} \\
 \dots
 \end{array}
 \quad \text{so } \begin{array}{l} x[-1] = -10 \\ x[-2] = -100 \\ x[-3] = -1000 \\ \text{etc.} \end{array}$$

NOTE THE REVERSED ORDERING