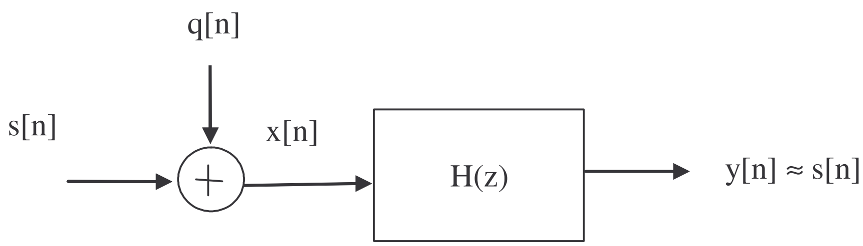
# ECGR 6114
# Computer Project: Wiener Filtering
# Student Name:___________



## Part1
## Overview




A Wiener filter is useful for stationary systems.
Consider the case of designing a filter H(z) to recover a signal s(n) from a corrupted signal x[n]=s[n]+q[n] where q[n] is noise. In the above adaptive filter formulation, the filter weights are not presumed constant.

As before, the filter is represented by its impulse response h[n] in the time domain, or by its Z-transform H(z) = B(z)/A(z) in the z-domain.

So,

H(z) = Y(z)/X(z) = B(z)/A(z)

The noncausal Wiener filter for stationary process is

Hncwiener(w) = S(w) / ( S(w)+Q(w) )

where S(w) is the power spectrum of the signal and
Q(w) is the power spectrum of the noise

The causal FIR wiener filter is set by autororrelation matrix

B= $\mathbf{R}^{-1}\,\mathbf{r}_{sx}$

Create a random binary (+/-1) signal s, for FIR filter of length Nb, with signal pulsewidth Nb/4, noise q with uniform pdf from -noise/2 to noise/2,

$N := 2^{15}$ $\qquad Nb := 4$ $\qquad pulsewidth := 32$

$noise := 2$ $\qquad signal := 1$

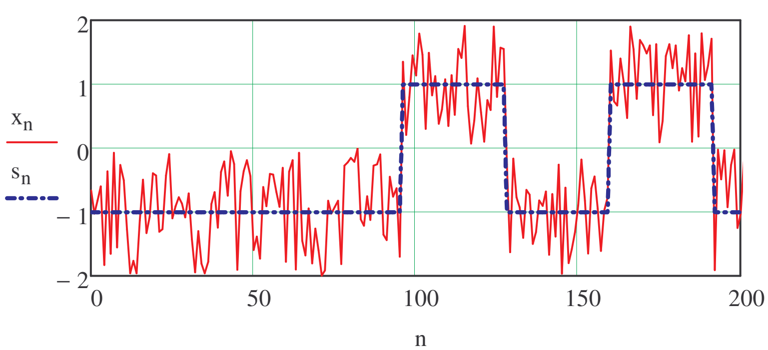
$n := 0..N-1$ $\qquad bb := 0..Nb$ $\qquad s_n := signal$ $\qquad pulsewidth = 32$

$$pp := 0..round\left(\frac{N}{pulsewidth}\right) + 10 \qquad ss_{pp} := rnd(signal) - \frac{signal}{2}$$

$$ss := 2 \cdot signal \cdot (rbinom(length(s), 1, 0.5) - 0.5)$$

$$s_n := ss_{trunc\left(\frac{n}{pulsewidth}\right)} \qquad q_n := rnd(noise) - \frac{noise}{2}$$

$$x_n := s_n + q_n$$




x =

| | 0 |
|---|---|
| 0 | -0.66 |
| 1 | -1.019 |
| 2 | -0.816 |
| 3 | -0.594 |
| 4 | -1.827 |
| 5 | -0.359 |
| 6 | -1.65 |
| 7 | -0.071 |
| 8 | -1.551 |
| 9 | -0.261 |
| 10 | -0.501 |
| 11 | -1.271 |
| 12 | -1.955 |
| 13 | -1.758 |
| 14 | -1.953 |
| 15 | ... |

**Q1**. What is the peak-peak voltage of the signal s?

**Q2**. What is the rms voltage of the noise q? (compute from experimental data q using a mathcad formula below)

**Q3**. What is the peak-peak voltage of the signal+noise signal x?

**Part 2**
## Frequency spectrum of the signals

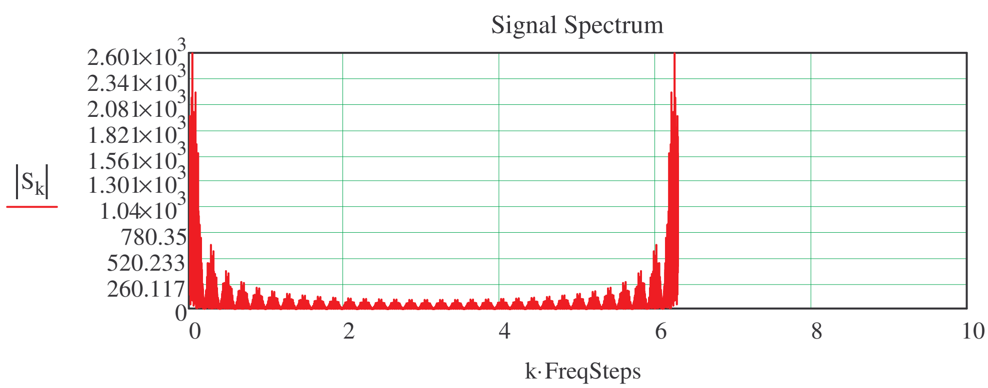
Now,
use the Mathcad CFFT function to take the Fast Fourier Transform (FFT) of your sampled data to get the frequency spectrum.
The CFFT is similar to DFT, except Mathcad version differs by a constant factor of N.
(To see definition of CFFT, see the Mathcad help page for CFFT).

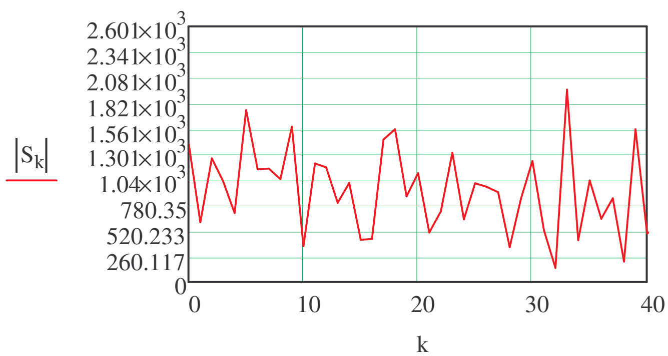
$Ts := 1$ $\qquad S := \text{rows}(s)\cdot\text{CFFT}(s)$ $\qquad S_0 = -1.408\times 10^3$ $\qquad N := \text{rows}(s)$

$FreqSteps := \frac{\pi\cdot 2}{N\cdot Ts}$ $\qquad FreqSteps = 1.917\times 10^{-4}$ $\qquad$ FreqSteps is frequency stepsize in FFT in rad/s.

$k := 0..N-1$

$smax := \max\left[\overrightarrow{(|S|)}\right]$ $\qquad smax = 2.601\times 10^3$ $\qquad N = 3.277\times 10^4$

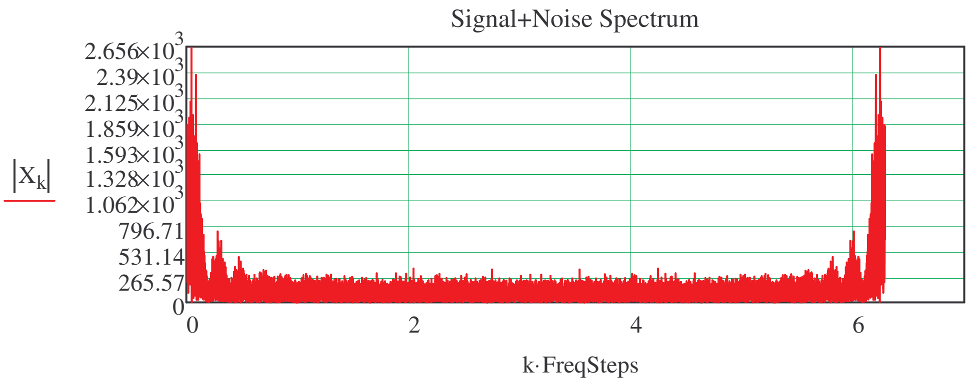



zoom in on a region

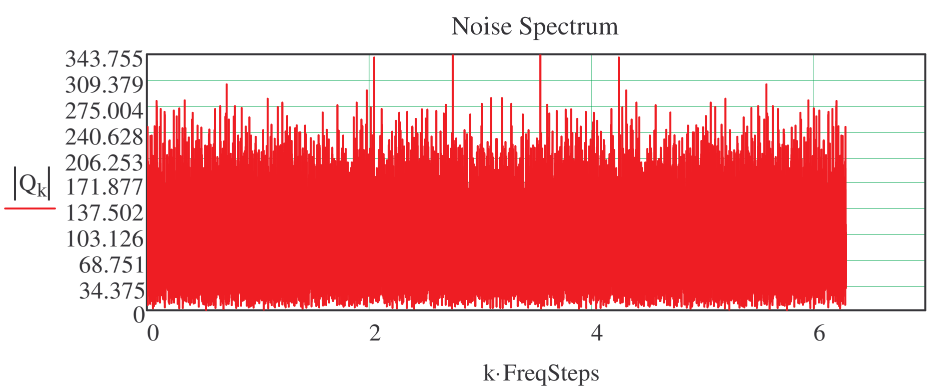
$X := \text{rows}(x)\cdot \text{CFFT}(x)$

$xmax := \max\left[\overrightarrow{(|X|)}\right]$ $\quad xmax = 2.656 \times 10^{3}$ $\quad N = 3.277 \times 10^{4}$




$Q := \text{rows}(q)\cdot \text{CFFT}(q)$

$qmax := \max\left[\overrightarrow{(|Q|)}\right]$ $\quad qmax = 343.755$



Smoothed spectrum (Computes Bartlett Method Averaged Periodigram)

$$\text{MagSqSmooth}(x,p) := \begin{array}{|l} sx \leftarrow \text{last}(x) \\ pp \leftarrow 2^{\text{trunc}(\log(p,2))} \\ rinc \leftarrow \dfrac{\text{rows}(x)}{pp} \\ \text{for } cc \in 0..rinc-1 \\ \quad YY_{cc} \leftarrow 0 \\ YcY \leftarrow \text{CFFT}(YY) \\ \text{for } cc \in 0..pp-1 \\ \quad \begin{array}{|l} xx \leftarrow \text{submatrix}(x, rinc\cdot cc, rinc\cdot cc + rinc - 1, 0, 0) \\ XX \leftarrow \text{rows}(xx)\cdot\text{CFFT}(xx) \\ XX \leftarrow \overrightarrow{\left[(|XX|)^2\right]} \\ YY \leftarrow XX + YY \end{array} \\ YY \leftarrow \dfrac{YY}{pp\cdot\text{rows}(YY)} \\ \text{return } YY \end{array}$$

p must be a power of 2

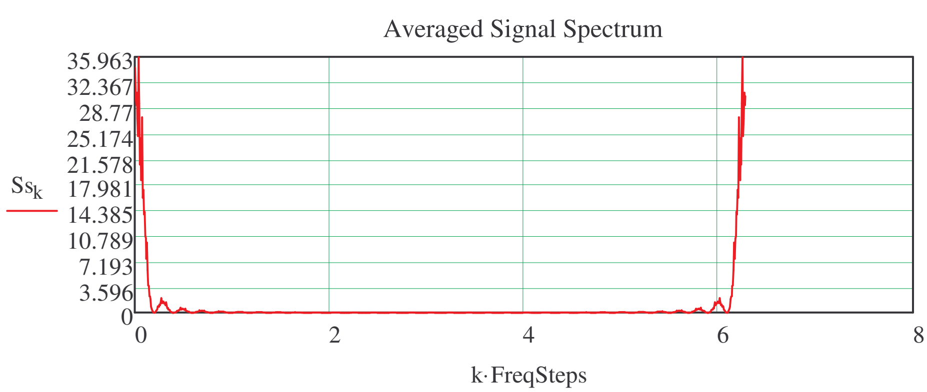
$\text{Smoo} := 32$

Smoothing factor equals number of spectra that are averaged

$\text{Ss} := \text{MagSqSmooth}(s, \text{Smoo})$

$\text{FreqSteps} := 2\cdot\dfrac{\pi}{\text{rows}(Ss)}$

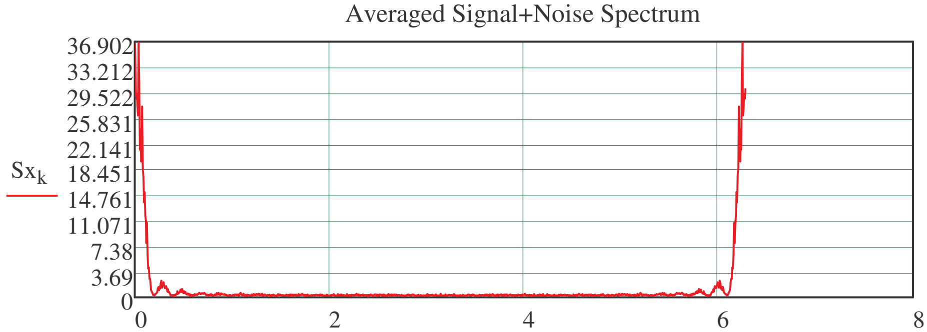
$\text{ssmmax} := \max\left[\overrightarrow{(|Ss|)}\right]$ $\qquad$ $\text{ssmmax} = 35.963$



$Sx := MagSqSmooth(x, Smoo)$

$xsmmax := \max\left[\overrightarrow{(|Sx|)}\right]$

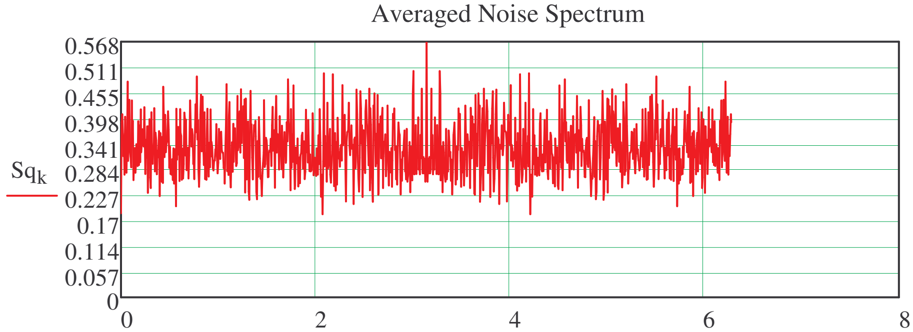
$xsmmax = 36.902$




$Sq := MagSqSmooth(q, Smoo)$

$qsmmax := \max\left[\overrightarrow{(|Sq|)}\right]$

$qsmmax = 0.568$




**Q5**. Compute (experimentally) the second moment of the noise, q, below. (compute from experimental data q using a mathcad formula below)

**Q5**. How does the value computed in Q5 compare to the magnitude of the above smoothed periodigram Sq of noise q (see Hayes Example 8.2.1).

## Part 3
## Non-causal Wiener filter

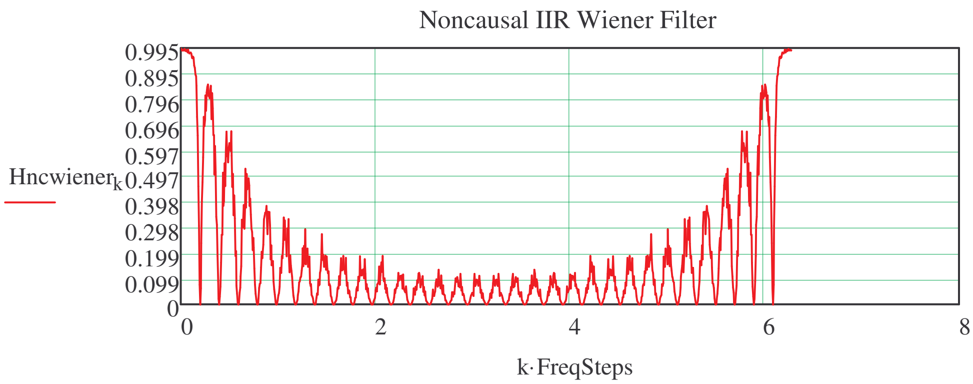
Noncausal Wiener filter for stationary process is

Hncwiener(w) = (S(w)/(S(w)+Q(w))

$$k := 0 .. \text{rows}(Ss) - 1$$

$$Hncwiener_k := \frac{Ss_k}{(Ss_k + Sq_k)}$$

$$hncwmax := \max\left[\overrightarrow{(|Hncwiener|)}\right] \qquad hncwmax = 0.995$$




**Q6**. Why does the Noncausal IIR Wiener filter reponse go to zero at certain frequencies?

yes/no

**Part 4**
**FIR Wiener filter**

The causal FIR wiener filter is set by autororrelation matrix

B= $\mathbf{R}^{-1}\,\mathbf{r}_{sx}$

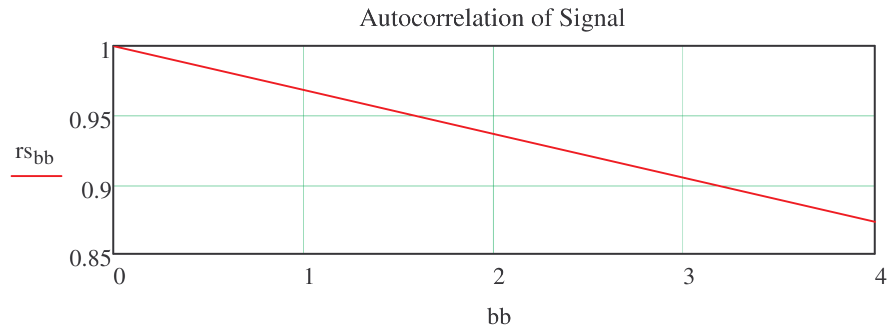
$$maxlag := Nb$$

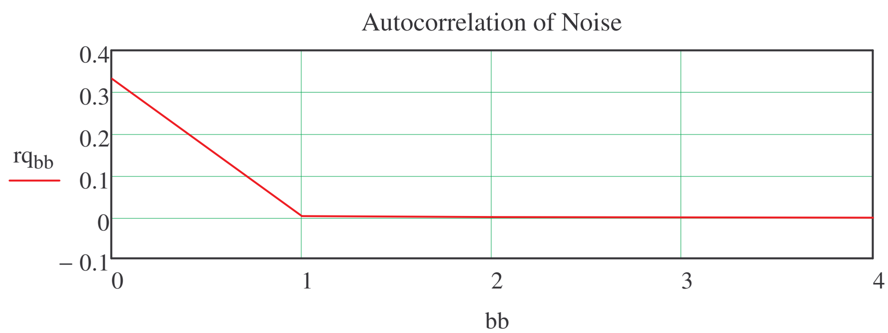
$$bb := 0..maxlag \qquad bbb := 0..maxlag$$

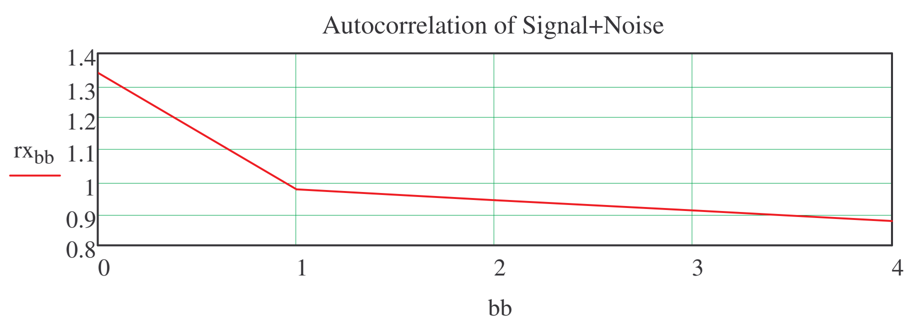
$$rx_{bb} := \left(\frac{1}{N-bb}\right)\cdot\sum_{k=bb}^{N-1}\left(x_k \cdot x_{k-bb}\right) \qquad rsx_{bb} := \left(\frac{1}{N-bb}\right)\cdot\sum_{k=bb}^{N-1}\left(s_k \cdot x_{k-bb}\right)$$

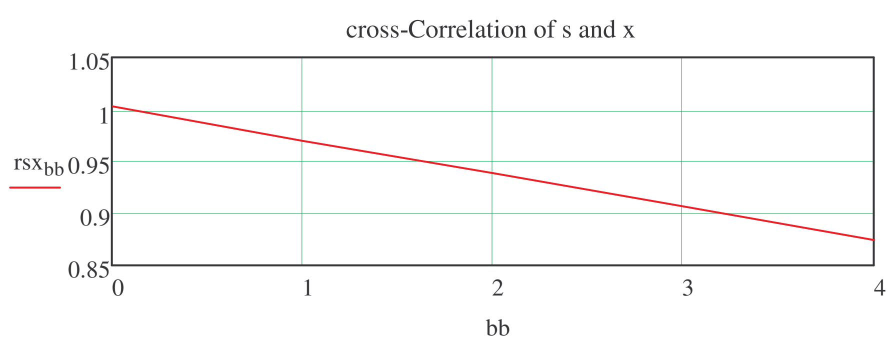
$$rs_{bb} := \left(\frac{1}{N-bb}\right)\cdot\sum_{k=bb}^{N-1}\left(s_k \cdot s_{k-bb}\right) \qquad rq_{bb} := \left(\frac{1}{N-bb}\right)\cdot\sum_{k=bb}^{N-1}\left(q_k \cdot q_{k-bb}\right)$$

$$rx = \begin{pmatrix} 1.34 \\ 0.976 \\ 0.942 \\ 0.91 \\ 0.877 \end{pmatrix} \qquad rsx = \begin{pmatrix} 1.003 \\ 0.97 \\ 0.939 \\ 0.907 \\ 0.875 \end{pmatrix} \qquad rs = \begin{pmatrix} 1 \\ 0.968 \\ 0.937 \\ 0.905 \\ 0.873 \end{pmatrix}$$


Autocorrelation of Signal
$rs_{bb}$
1
0.95
0.9
0.85
0
1
2
3
4
bb


Autocorrelation of Noise
$rq_{bb}$
0.4
0.3
0.2
0.1
0
− 0.1
0
1
2
3
4
bb


Autocorrelation of Signal+Noise
$rx_{bb}$
1.4
1.3
1.2
1.1
1
0.9
0.8
0
1
2
3
4
bb


cross-Correlation of s and x
$rsx_{bb}$
1.05
1
0.95
0.9
0.85
0
1
2
3
4
bb

**Q7**. Refer to page 5 of the Wiener handout and to the plots above; does rsx = rs?
yes/no

**Q8**. Refer to page 5 of the Wiener handout and to the plots above; does rx = rs + rq?
yes/no

compure autocorrelation matrix of signal plus noise

$$R_{bb,bbb} := \text{if}\left(bb - bbb > 0, rx_{bb-bbb}, rx_{bbb-bb}\right)$$

$$R = \begin{pmatrix} 1.34 & 0.976 & 0.942 & 0.91 & 0.877 \\ 0.976 & 1.34 & 0.976 & 0.942 & 0.91 \\ 0.942 & 0.976 & 1.34 & 0.976 & 0.942 \\ 0.91 & 0.942 & 0.976 & 1.34 & 0.976 \\ 0.877 & 0.91 & 0.942 & 0.976 & 1.34 \end{pmatrix}$$

**Q9**. Is the autocorrelation matrix R above a Toeplitz matrix?
yes/no

**Q10**. Is the autocorrelation matrix R above a Hermitian matrix?
yes/no

Compute FIR Filter

$$\mathrm{Bwiener} := R^{-1}\cdot \mathrm{rsx} \qquad \mathrm{Bwiener} = \begin{pmatrix} 0.348 \\ 0.224 \\ 0.153 \\ 0.109 \\ 0.086 \end{pmatrix}$$

The error is

$$E := rs_0 - \left(\overline{\mathrm{rsx}}\right)^T R^{-1}\cdot \mathrm{rsx} \qquad E = 0.116$$

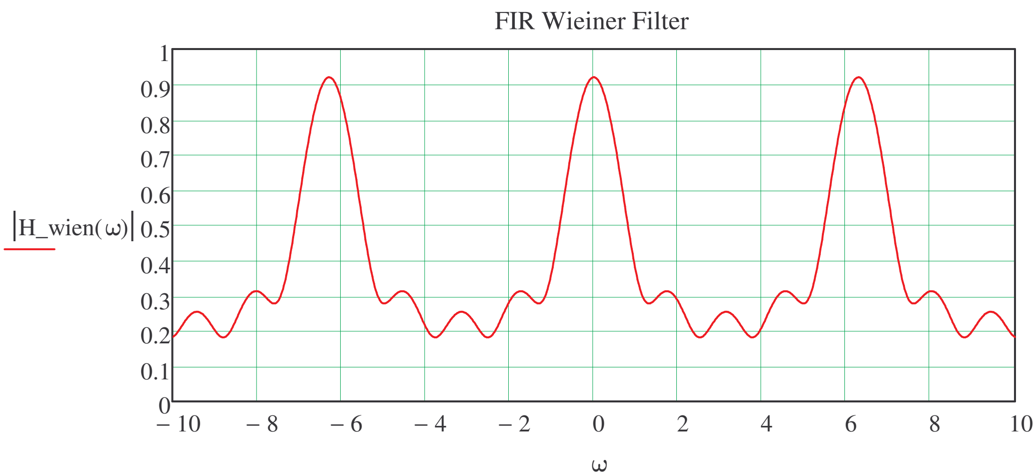
Finally, we have Hbilin(z)=B(z)/N(z)

$$\mathrm{Hwien}(\mathrm{num}, z) := \sum_{k=0}^{\mathrm{rows(num)}-1} \left(\mathrm{Bwiener}_k \cdot z^{-k}\right)$$

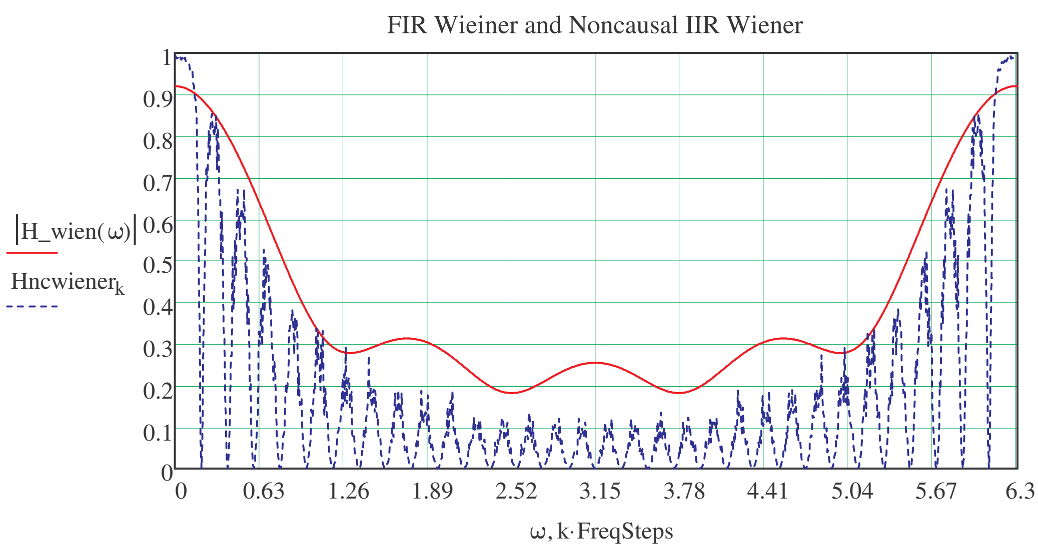
Plot the frequency reponse of the filter

$$H\_wien(\omega) := Hwien\left(Bwiener, e^{i\cdot\omega}\right)$$

$Nb = 4$ $signal = 1$ $noise = 2$ $pulsewidth = 32$ $N = 3.277 \times 10^{4}$




Comparison of filters

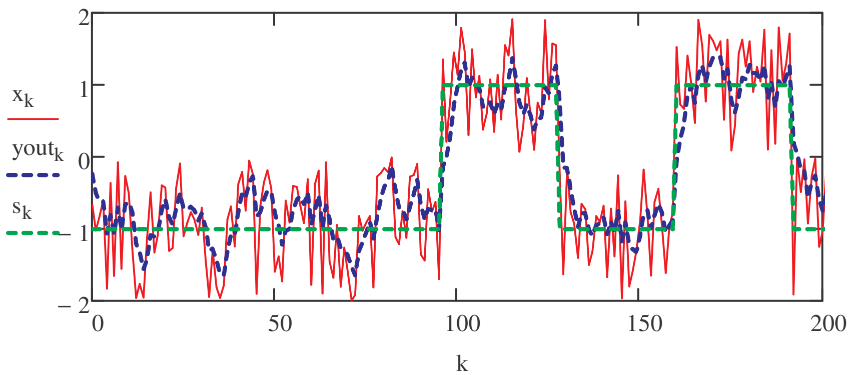



**Q11**. Does the frequency response of the FIR Wiener filter resemble the reponse of the Noncausal Wiener filter (in terms of the overall tracking of peaks)?

yes/no

## Part 5
## FIR Wiener filtered signal

$$yout_n := \sum_{k=0}^{rows(Bwiener)-1} \left[Bwiener_k \cdot if\left[(n-k) < 0, 0, x_{n-k}\right]\right]$$




**Q12**. Increase the order Nb of the FIR Wiener filter to 16, and replot all results above. Use these new plots in the report that you turn in, but do not change the answers to the earlier questions

**Q13**. For the filtered signal, Is the noise reduced without greatly distorting the signal? yes/no.