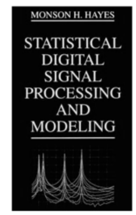
EEGR 6114
Digital Signal Processing II

Introduction



MOSAIC Accounts

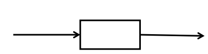
- If you do not have a MOSAIC computer account, see TA in computer lab to obtain MOSAIC computer account

- Syllabus

DSP

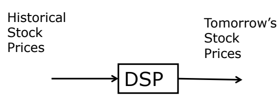
- Why?
  - Digital Circuit speed/cost/density?
  - Analog Circuit variation/Tolerances?
  - Advances in algorithms(FFT/Filters)?
  - DSP Chips/HDTV apps./Coding?
- Why not?
  - Still frequency limits?
  - Where do you store 1 GByte/Second

- What is a filter? What is a system?
- Impulse/freq response? Convolution?
- Is there naturally digital data?
- Why do you use a Butterworth, Elliptic?
- Does the signal/system change with time?
- How do you adapt to change?

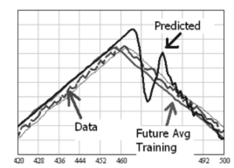


Good DSP II Design Problem

- What type of Butterworth filter is needed below?
- Will Butterworth work?
- Will solution change day to day? Stock-to-stock?
- Does the data determine the filter?



Example Predictor

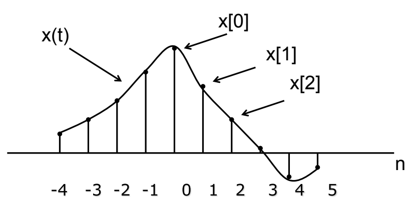


Discrete-Time Signals

In sampled system, x [n]= x(n Ts)

[ ] Square bracket

( ) parentheses



NOTE: data is often digital to begin with!

Sequences

- What is a sequence?
  An order set, order counts
- Mathematically, a sequence is best seen as a function x:

$$\boxed{x : Z \to R}$$

where above notation indicates "x maps Z into R"

Domain = Z = Integers (….-1,0,1,…)

Range = Real Numbers =R

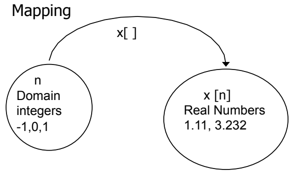
OR

simply denoted as x [n],

where square brackets [ ] implies $n \in Z$

Sequences

Mapping



Table

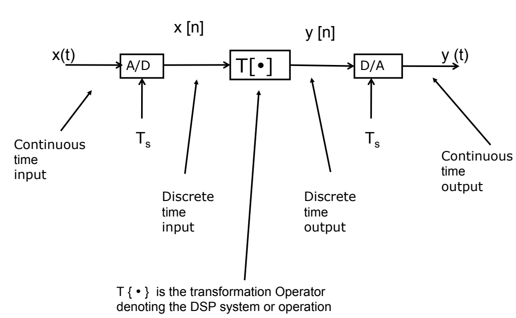
n	x [n]
-2	2.2
-1	1.8
0	4
1	2.3
2	3.7

Sequence
(ordered set)

X= {x [n]} ={ ….x[-1], x[0], x[1], …. }

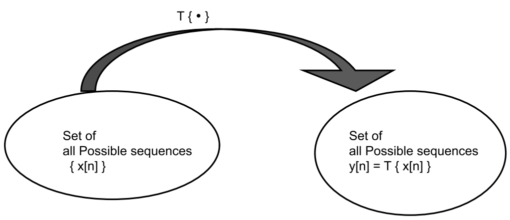
= ….,X[-1],X[0],X[1],….

Discrete-time systems



T{ }

- y[n] = T { x [n] }
  where T { • } is transformation or operator or function relating discrete-time input to discrete-time output
- Transformation/ Rule/ Mapping/ Function

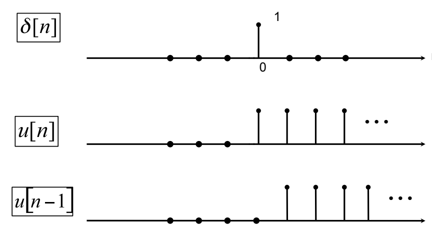


Important Discrete-Time Signals

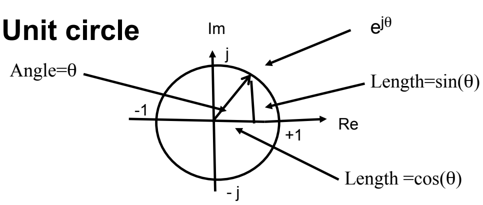
In sampling a continuous-time signal,
x [n]= x(n Ts)

Unit sample, or impulse, or Kronecker delta

Unit step

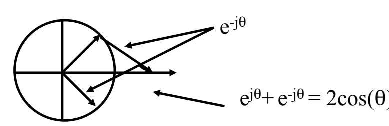


Unit circle



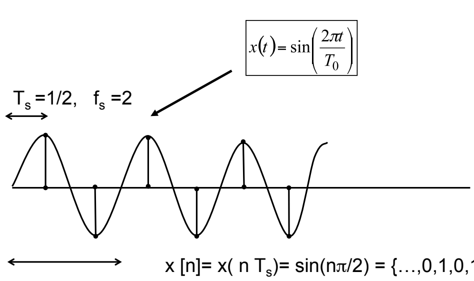
- $e^{j\theta} = \cos(\theta) + j\sin(\theta) = \alpha + j\beta$
- Can be viewed as the point in complex plane with coordinates (cos θ, sin θ)
- Can also be viewed as a vector, as shown above
- $|e^{j\theta}|$=?

Unit Circle



- As shown above, vector viewpoint is useful when adding
- Phasor notation can also be useful in polynomials
- Consider finding square root of j
  $(\gamma\angle\theta)^2 = \gamma^2 \angle 2\theta = j = 1 \angle 90^0$
  or $(\gamma e^{j\theta})^2 = \gamma^2 e^{j2\theta} = j = 1\ e^{j\pi/2}$ (assuming γ is positive real)
- So, there are 2 roots to 2nd order polynomial
  $1\angle 45^o$ AND $1\angle 225^o$

Plot of sampled sinusoid



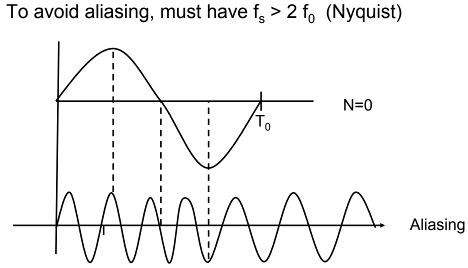
$x[n] = x(nT_s) = \sin(n\pi/2) = \{\ldots,0,1,0,1,0,\ldots\}$

$\omega_0 = 2\pi(f_0/f_s)$ = ? rad/sample

How many radians between each sample?

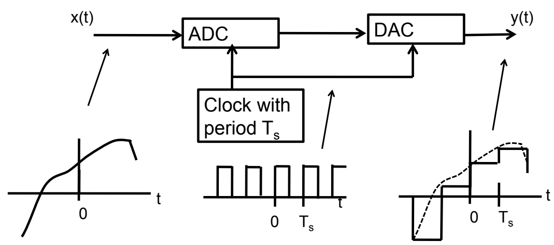
Aliasing

- Consider an analog sinusoid with extra 2π between samples
- Samples are exactly the same
- So, two different analog signals give rise to same samples
- To avoid aliasing, must have $f_s > 2 f_0$ (Nyquist)



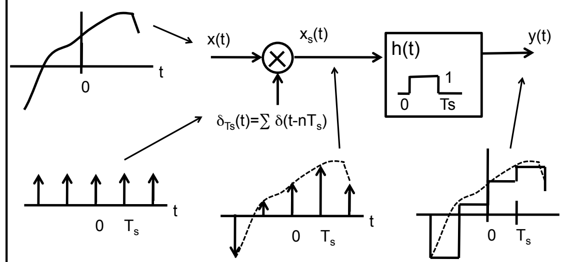
A/D & D/A Conversion

- ADC: Analog-to-Digital Converter
- DAC: Digital-to-Analog Converter
- Consider the ADC/DAC system below (simple A-to-D-to-A system)
- How do we model this mathematically?



Sampling (Model of A/D+D/A Conversion)

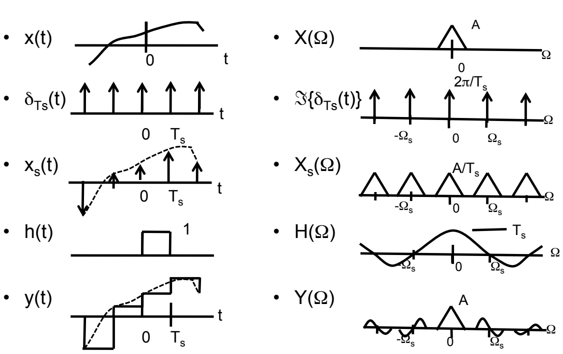
- Consider ADC model below, where $h(t) = rect[(t-T_s/2) / T_s]$
- $rect(t/\tau)$ is rectangular pulse centered at zero of width τ and height 1
- Below, $x_s(t)$ is the sampled signal
- This model has the same input/output as the ADC+DAC system



Sampling

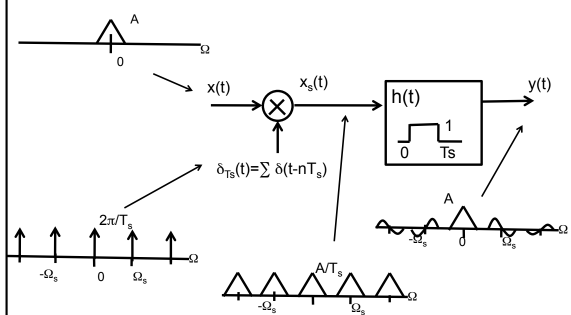
$rect(t/T_s) \Leftrightarrow T_s \, \text{sinc}(\Omega T_s/2)$

$g_1(t)g_2(t) \Leftrightarrow \frac{1}{2\pi} G_1(\Omega) * G_2(\Omega)$



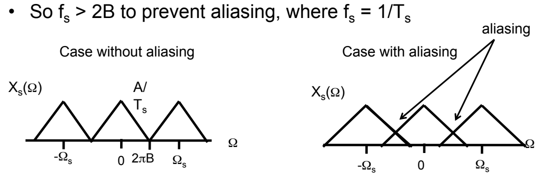
Sampling Redrawn Using Spectra

- This model has the same input/output as the ADC+DAC system
- h(t)=rect{(t-Ts/2)/(Ts)},



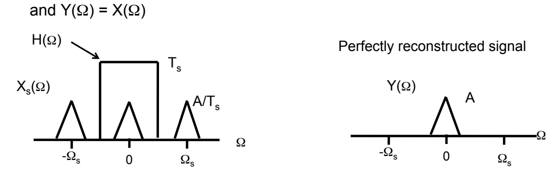
Nyquist Rate and Aliasing

- Consider the spectrum of the sampled signal $X_s(\Omega)$ below
- If Ω_s is too small, the spectra will overlap, and information will be lost
- This overlap is aliasing
- The minimum sampling rate tp prevent aliasing is the Nyquist rate, 2B, where B is the bandwidth of the signal x(t)
- So $f_s > 2B$ to prevent aliasing, where $f_s = 1/T_s$



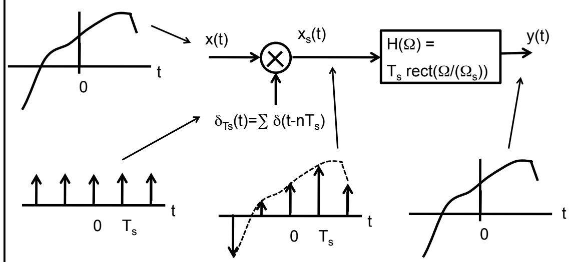
Perfect Reconstruction Filter

- The filter that modeled the ADC-DAC pair had $h(t) = rect[(t-T_s/2) / T_s]$, and is referred t as a zero-order hold filter
- This filter had the effect of distorting the time domain signal by changing the smooth x(t) into a "staircase/stepwise" approximation y(t)
- In the frequency domain, H(Ω) multiplied the signal spectrum by a sinc() function, and did not remove the harmonics from the spectrum
- Below, if h(t)is an ideal lowpass filter with $H(\Omega) = T_s \, rect(\Omega/(\Omega_s))$, then the output of the system will exactly equal the original input; y(t) = x(t) and Y(Ω) = X(Ω)



Perfect Reconstruction System

- Consider model below, where $H(\Omega) = T_s \, rect(\Omega/(\Omega_s))$
- Now the output signal y(t) exactly equals the input signal x(t)

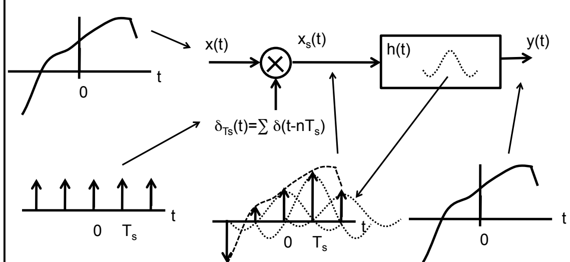


Sampling Theorem

- Nyquist-Shannon Sampling theorem
  - If a signal is strictly bandlimited to B Hz, then it can be perfectly reconstructed from its samples, if the signal is sampled at sampling rate greater than 2B Hz.
- This means that all time points between the sample points can be exactly recovered
- This theorem is essentially the same result as the perfect reconstruction systems given in the previous slides, since y(t) = x(t) !!

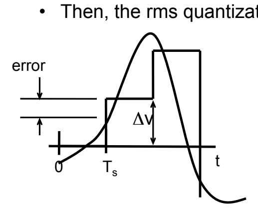
Perfect Reconstruction (Convolution View)

- The impulse response of the perfect reconstruction filter is $h(t)=sinc(\pi t/T_s)$
- When convolved with $x_s(t)$, the sinc() functions interpolate between samples as illustrated below



RMS Quantization Noise

- When an ADC with voltage step size Δv digitizes a signal m(t), the difference between the analog input signal and nearest digital signal level output acts as if a random quantization noise error q(t) was added to the analog signal m(t)
- Then, the rms quantization noise voltage is $V_{q\ rms}$



(Section 4.8.3)

$$V_{q\ rms} = \frac{\Delta v}{\sqrt{12}}$$