CUMULATIVE DISTRIBUTION FUNCTION (CDF)

p. 61

$F_x(\alpha) = P(x \leq \alpha)$ = PROBABILITY $x \leq \alpha$

MOST COMMONLY WRITTEN $F_x(x)$

PROPERTIES:

$0 \leq F_x(\alpha) \leq 1$

$F_x(\alpha) - F_x(\beta) = P(\beta < x \leq \alpha)$

$F_x(-\infty) = 0$, $F_x(\infty) = 1$

PROBABILITY DENSITY FUNCTION (pdf)

$$p_x(x) = \frac{d F_x(x)}{dx} \quad \text{SO} \quad F_x(x) = \int_{-\infty}^{x} p_x(\alpha)\, d\alpha$$

EXPECTED VALUE

p. 63

$$E[f(x)] = \int_{-\infty}^{\infty} f(x)\, p_x(x)\, dx$$

$\mu_x = E[x]$ = MEAN $\quad E[x^2]$ = SECOND MOMENT

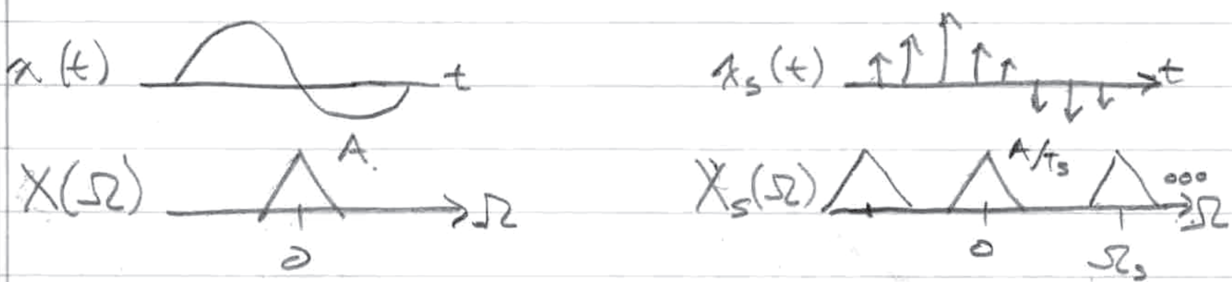
$E[(x-\mu)^2] = 2^{ND}$ CENTRAL MOMENT $= \sigma_x^2 = E[x^2] - \mu^2$

QUANTIZATION NOISE

$E[v^2] = \frac{\Delta^2}{12}$ AND $\mu_v = 0$

RMS NOISE $= \Delta / \sqrt{12}$

Discrete Time Fourier Transform (DTFT)



$$x(t) \longrightarrow \otimes \longrightarrow x_s(t)$$

$$\uparrow \delta_{T_s}(t) = \sum_{-\infty}^{\infty} \delta(t - nT_s)$$

$$X_s(\Omega) = \mathcal{F}\{x_s(t)\} = \int_{-\infty}^{\infty} x_s(t) e^{-j\Omega t} \, dt$$

$$= \int_{-\infty}^{\infty} x(t) \left(\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right) e^{-j\Omega t} \, dt$$

$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} x(t) \, \delta(t - nT_s) e^{-j\Omega t} \, dt$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) e^{-jn\Omega T_s}$$

<u>DTFT</u> p. 13

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{jn\omega} \, d\omega$$

<u>By Comparison:</u>

$$X(\omega) = X_s(\Omega) \Big|_{\Omega = \omega / T_s}$$

Convolution

For a linear time-invariant (LTI) system

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Also note

$$x[n] * h[n] = h[n] * x[n]$$

And

$$\text{DTFT}\{x[n] * h[n]\} = X(\omega) H(\omega)$$

Eigenfunctions of LTI systems

Suppose input to LTI system is $e^{j\omega n}$

$$x[n] = e^{j\omega n} \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

then

$$y[n] = x[n] * h[n] = h[n] * x[n] = h[n] * e^{j\omega n}$$
$$= \sum_{k=-\infty}^{\infty} h[k]\, e^{j\omega(n-k)}$$

$$= e^{j\omega n} \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\omega k}$$

$$= \underbrace{e^{j\omega n}}_{\text{Eigenfunction}} \; \underbrace{H(\omega)}_{\text{Eigenvalue}}$$

$e^{j\omega n}$ is eigenfunction of LTI systems with convolution relation. → hence importance of frequency domain for LTI systems.

DFT Discrete Fourier Transform

p. 18

- FFT (Fast Fourier Trans.) is just fast DFT

DFT is just samples of DTFT

So DFT is sampled in both time & frequency

So $X[k] = X(\omega)\big|_{\omega = k2\pi/N}$ if x[n]=0 outside n=0->N-1

Then $\text{DFT}\{x[n]\} = X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi}{N}\right)nk}$

And Inverse $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\left(\frac{2\pi}{N}\right)nk}$.

Note! - Summations are finite: $0 \rightarrow N-1$

- So implies data is finite length

Periodicity

- Both $X[k]$ and $x[n]$ are periodic
- Period is N

Circular Shift

- Since $x[n]$ is periodic, shifts are circular
- Shift by n_0 becomes shifts by $n_0 \bmod N$
- $((n_0))_N$ = $n_0 \bmod N$ = n_0 modulo N
- $((n_0))_N = \alpha$ such that $\alpha + kN = n_0$, $0 \le \alpha \le N-1$

Circular Convolution

p. 20

$$x[n] \circledN h[n] = \sum_{k=0}^{N-1} x[k] h[((n-k))_N]$$

$$\text{DFT}\{x[n] \circledN h[n]\} = X[k] H[k]$$