Notes on $|H(\Omega)|^2$, $|H(s)|^2$, $|H(z)|^2$, and $|H(\omega)|^2$

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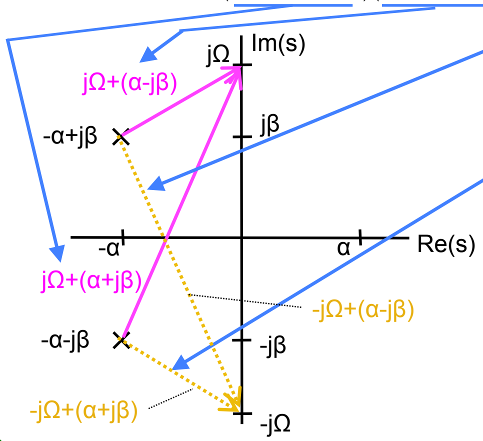
Graphical Interpretation of $|H(\Omega)|^2$ from H(s)

$$H(s)=\frac{1}{(s+(\alpha+j\beta))(s+(\alpha-j\beta))} \rightarrow H(\Omega)=\frac{1}{(j\Omega+(\alpha+j\beta))(j\Omega+(\alpha-j\beta))}$$

$$|H(s)|^2=\frac{1}{(s+(\alpha+j\beta))(s+(\alpha-j\beta))(s+(\alpha+j\beta))^*(s+(\alpha-j\beta))^*}$$

$$|H(\Omega)|^2=\frac{1}{(j\Omega+(\alpha+j\beta))(j\Omega+(\alpha-j\beta))(-j\Omega+(\alpha-j\beta))(-j\Omega+(\alpha+j\beta))}$$



- H(s) has 2 stable conjugate poles on left
- $|H(s)|^2$ has conjugate operator leading to –jΩ terms in denominator
- The denominator terms are drawn as vectors
- Phases are opposites and cancel; net phase=0

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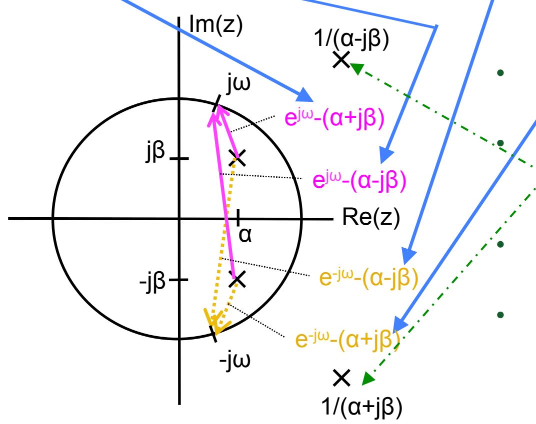
Graphical Interpretation of $|H(\omega)|^2$ from H(z)

$$H(z)=\frac{1}{(z-(\alpha+j\beta))(z-(\alpha-j\beta))} \rightarrow H(\omega)=\frac{1}{\left(e^{j\omega}-(\alpha+j\beta)\right)\left(e^{j\omega}-(\alpha-j\beta)\right)}$$

$$H(z)H^*\left(1/z^*\right)=\frac{1}{(z-(\alpha+j\beta))(z-(\alpha-j\beta))\left(1/z^*-(\alpha+j\beta)\right)^*\left(1/z^*-(\alpha-j\beta)\right)^*}$$

$$|H(\omega)|^2=\frac{1}{\left(e^{j\omega}-(\alpha+j\beta)\right)\left(e^{j\omega}-(\alpha-j\beta)\right)\left(e^{-j\omega}-(\alpha-j\beta)\right)\left(e^{-j\omega}-(\alpha+j\beta)\right)}$$



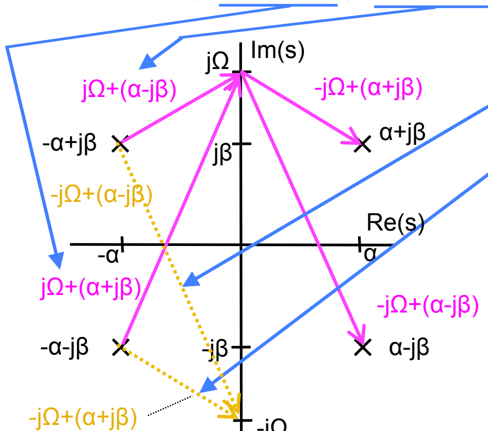
- H(z) has 2 stable conjugate poles inside unit circle
- H(z) H*(1/z*) has 4 poles shown, 2 reciprocal poles arising from H*(1/z*)
- The denominator terms are drawn as vectors
- Phases are opposites and cancel; net phase=0

Relation of $|H(\Omega)|^2$ and $|H(s)|^2$ and $|H(s)H^*(-s^*)|$

$$\left|H(s)H^*(-s^*)\right|=\frac{1}{|s+(\alpha+j\beta)||s+(\alpha-j\beta)|}\quad\frac{1}{\left|\left(-s^*+(\alpha+j\beta)\right)^*\right|\left|\left(-s^*+(\alpha-j\beta)\right)^*\right|}$$

$$|H(s)|^2=\frac{1}{(s+(\alpha+j\beta))(s+(\alpha-j\beta))(s+(\alpha+j\beta))^*(s+(\alpha-j\beta))^*}$$

$$|H(\Omega)|^2=\frac{1}{(j\Omega+(\alpha+j\beta))(j\Omega+(\alpha-j\beta))(-j\Omega+(\alpha-j\beta))(-j\Omega+(\alpha+j\beta))}$$



- H(s)H(-s) has 4 poles shown, 2 being the stable poles on the left
- $|H(s)|^2$ has double poles at each of the 2 left side poles
- Note $|H(s)|^2=|H(s)H^*(-s^*)|$ for s=jΩ *along jΩ axis only*, by comparing the lengths of the vectors