

Name: _____



Last 4 digits of student Number: _____

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Do NOT begin until told to do so, you may NOT leave the room during the exam
 Make sure that you have all pages before starting
 Open book (text or Weldon or both), 2 sheets front/back notes, 1 Smith chart with markings, BASIC CALCULATOR, no other electronic devices you may bring a compass and ruler
 DO ALL WORK IN THE SPACE GIVEN, DO NOT LEAVE THE ROOM DURING EXAM
 Do NOT use the back of the pages, do NOT turn in extra sheets of work/paper
 Multiple-choice answers should be within 10% of correct value. Show ALL work, even for multiple choice

ACADEMIC INTEGRITY:

Students have the responsibility to know and observe the requirements of The UNCC Code of Student Academic Integrity. This code forbids cheating, fabrication or falsification of information, multiple submission of academic work, plagiarism, abuse of academic materials, and complicity in academic dishonesty

Unless otherwise noted:

$[x \ y \ z]^T$ denotes column vector of coordinates x,y,z

$F\{\}$ denotes Fourier transform {or Fourier series, as implied in problem} $F^{-1}\{\}$ denotes inverse Fourier

ω denotes frequency in rad/second, f denotes frequency in Hz

* denotes convolution

$x^*(t)$ denotes the conjugate of $x(t)$

Useful constants, etc:

$e \approx 2.72$

$1/e \approx 0.37$

$\ln(2) \approx 0.69$

$\log_{10}(2) \approx 0.30$

$\log_{10}(10) \approx 1.0$

$\log_{10}(e) \approx 0.43$

$\pi \approx 3.14$

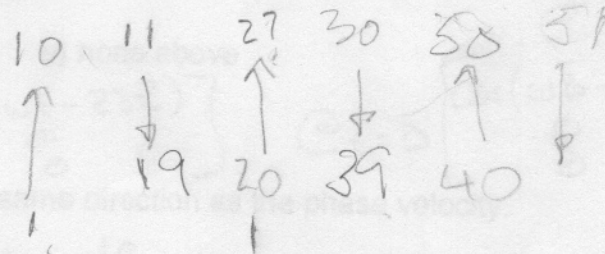
$\sqrt{2} \approx 1.41$

$\ln(4) \approx 1.38$

$\log_{10}(3) \approx 0.48$

$\log_{10}(0.1) \approx -1$

$\cos(\pi/4) \approx 0.71$



$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$

$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$

$\mathbf{A} \cdot \mathbf{A} = |\mathbf{A}|^2$

$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$

$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$

$\mathbf{A} \times \mathbf{A} = 0$

$\cos(A) \cos(B) = 0.5 \cos(A - B) + 0.5 \cos(A + B)$

$\sin(A) \cos(B) = 0.5 \sin(A - B) + 0.5 \sin(A + B)$

$e^{j\theta} = \cos(\theta) + j \sin(\theta)$

$\sin(A) \sin(B) = 0.5 \cos(A - B) - 0.5 \cos(A + B)$

$\delta(t) \leftrightarrow 1$

$\Pi(t/\tau) \leftrightarrow \tau \cdot \text{sinc}(\pi f \tau)$

$\Delta\left(\frac{t}{\tau}\right) \leftrightarrow \frac{\tau}{2} \cdot \text{sinc}^2(\pi f \tau / 2)$

$e^{j2\pi f_0 t} \leftrightarrow \delta(f - f_0)$

$e^{-at}u(t) \leftrightarrow \frac{1}{a + j2\pi f}$

$e^{-a|t|} \leftrightarrow \frac{2a}{a^2 + (2\pi f)^2}$

$e^{-t^2/(2\sigma^2)} \leftrightarrow \sigma\sqrt{2\pi} e^{-2(\sigma\pi f)^2}$

$1 \leftrightarrow \delta(f)$

$\text{sgn}(t) \leftrightarrow \frac{1}{j\pi f}$

$\cos(2\pi f_0 t) \leftrightarrow 0.5(\delta(f + f_0) + \delta(f - f_0))$

$\sin(2\pi f_0 t) \leftrightarrow 0.5j(\delta(f + f_0) - \delta(f - f_0))$

$2B \text{sinc}(2\pi Bt) \leftrightarrow \Pi(f/(2B))$

$B \text{sinc}^2(\pi Bt) \leftrightarrow \Delta(f/(2B))$

$a \sin(x) + b \cos(x) = \sqrt{a^2 + b^2} \sin(x + \arctan(-b/a))$

5 Points Each, Circle the best answer

1. The wavelength of a 1 GHz plane wave in vacuum is $\lambda = c/f = 3 \times 10^8 / 10^9$

- a) 0.3 m b) 0.5 m c) 2 m d) 3 m e) none above

2. The length of time for a 1 GHz plane wave in vacuum to travel a distance of 1 meter is

- a) 0.33 μ s b) 3.3 μ s c) 1 ns d) 3.3 ns e) none above

$$t = \frac{d}{v} = \frac{1}{c} = \frac{1}{3 \times 10^8} = \frac{1}{0.3 \times 10^9} = 3.33 \times 10^{-9}$$

3. In a plane wave, the direction of the E-field is orthogonal to the H-field at all points in space with non-zero fields.

- a) True b) False

4. The direction of propagation of a plane wave in vacuum with phasor $\mathbf{E} = [1 \ 0 \ 0]^T e^{-j23z}$ is

- a) +x b) -y c) +z d) -z e) none above

$$\text{Re}\{E\} = \text{Re}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{j\omega t} e^{-j23z} \right\} = \begin{bmatrix} \cos(\omega t - 23z) \\ 0 \\ 0 \end{bmatrix} \quad @ t = \Delta \quad \begin{bmatrix} \cos(\omega \Delta - 23z) \\ 0 \\ 0 \end{bmatrix}$$

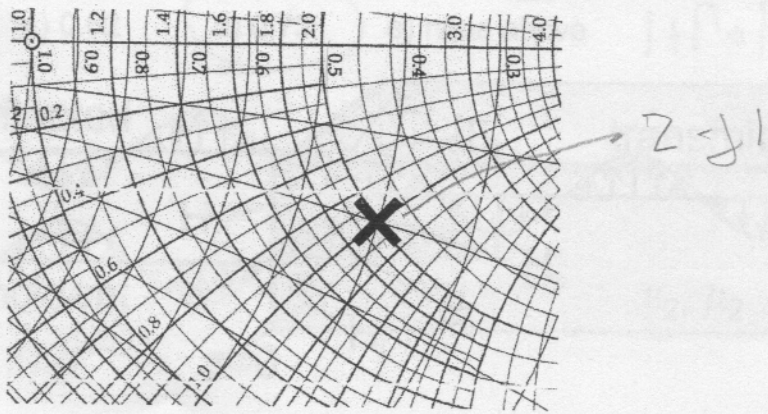
$z = \frac{\omega \Delta}{23}$

5. In a backward wave, the group velocity is in the same direction as the phase velocity.

- a) True b) False *opposite*

6. Given the normalized Smith chart of measured antenna impedance at 100 MHz in a $Z_0 = 50$ ohm system, with the antenna impedance marked by the "x" below, the antenna impedance (not normalized) in ohms is

- a) 100 + j50 b) 50 - j100 c) 100 - j20 d) 100 - j50 e) none above



5 Points Each, Circle the best answer

7. For a time-varying field in vacuum with oblique incidence at the boundary with a conductor, the tangential E-field at the boundary must equal zero

- a) True b) False

8. For a 1 GHz plane wave in glass having $\sigma=0$, $\mu=\mu_0$, $\epsilon=3\epsilon_0$, the propagation constant $\gamma =$

- a) $33 + j43$ b) $j36$ c) $j47$ d) 63 e) none above

$$\gamma = j\omega\sqrt{\mu\epsilon} = j\omega\sqrt{\mu_r\epsilon_r}\epsilon_0 = j\frac{2\pi \cdot 10^9 \sqrt{1 \cdot 3}}{3 \times 10^8} = j\frac{2\pi \cdot 10\sqrt{3}}{3} \approx j36.3$$

9. For a 1 GHz plane wave propagating in glass having $\sigma=0$, $\mu=\mu_0$, $\epsilon=3\epsilon_0$, the intrinsic impedance in ohms is $\eta =$

- a) $121 + j 53$ b) 169 c) 189 d) 218 e) none above

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r}{\epsilon_r}} \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \sqrt{\frac{\mu_r}{\epsilon_r}} = 377 \sqrt{\frac{1}{3}} = 218$$

10. For a 1 GHz plane wave propagating in glass having $\sigma=0$, $\mu=\mu_0$, $\epsilon=3\epsilon_0$, the wavelength is $\lambda =$

- a) 0.13 m b) 0.17 m c) 0.55 m d) 0.77 m e) none above

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{36.3} = 0.17$$

11. For a 1 GHz plane wave in vacuum with normal incidence into glass having $\mu=\mu_0$, $\epsilon=3\epsilon_0$, the reflection coefficient is $\Gamma =$

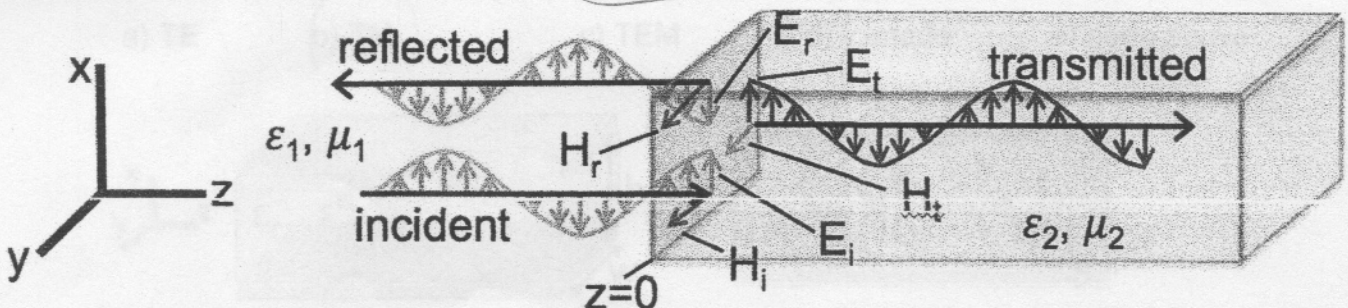
- a) -0.27 b) -0.38 c) -0.52 d) $0.62 + j0.21$ e) none above

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{218 - 377}{218 + 377} = \frac{-159}{595} = -0.27$$

12. As shown below, for a 2 GHz plane wave in vacuum with normal incidence into glass having $\mu_2=\mu_0$, $\epsilon_2=3\epsilon_0$, the ratio of the transmitted E-field E_t to the incident E-field E_i is $E_t/E_i =$

- a) -0.55 b) 0.55 c) 0.62 d) 0.73 e) none above

$$1 + \Gamma = 1 - 0.27 = 0.73$$



5 Points Each, Circle the best answer

13. The plane wave E-field with phasor $\mathbf{E} = [0 \ 2e^{-j\pi} \ 0]^T e^{-j50\omega z}$ has linear polarization.

- a) True b) False

14. The critical angle in degrees for total internal reflection for two media with $\mu_1 = \mu_0$, $\epsilon_1 = 5.2\epsilon_0$ and $\mu_2 = \mu_0$, $\epsilon_2 = \epsilon_0$ is $\theta_c =$

- a) 26 b) 34 c) 46 d) 56 e) none above

$$\text{Arctan} \frac{n_2}{n_1} = \text{Arctan} \left(\sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}} \right) = \text{Arctan} \left(\sqrt{\frac{1 \cdot 1}{1 \cdot 5.2}} \right) = \text{Arctan}(0.44) = 0.41 \text{ rad} = 24.4^\circ$$

15. The average power density in W/m^2 equals $0.5 \text{Re}\{\mathbf{E} \times \mathbf{H}^*\}$ where E-field phasor \mathbf{E} is in V/m , and the H-field phasor \mathbf{H} is in A/m .

- a) True b) False

16. The TEM impedance in ohms of a parallel-plate line of width $w = 8 \text{ mm}$ on a 1 mm thick printed circuit board having $\mu = \mu_0$, $\epsilon = 3.8\epsilon_0$, is $Z_0 =$

- a) 18 b) 24 c) 32 d) 48 e) none above

$$\frac{h}{w} \sqrt{\frac{\mu}{\epsilon}} = 377 \frac{h}{w} \sqrt{\frac{\mu_r}{\epsilon_r}} = 377 \cdot \frac{1}{8} \cdot \sqrt{\frac{1}{3.8}} = 24.2$$

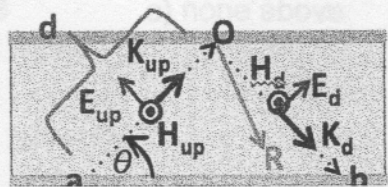
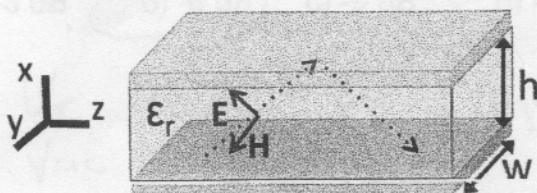
17. The TE_1 cutoff frequency of a parallel-plate waveguide of width $w = 8 \text{ mm}$ on a 1 mm thick printed circuit board having $\mu = \mu_0$, $\epsilon = 3.8\epsilon_0$, is $f_c =$

- a) 19 GHz b) 38 GHz c) 44 GHz d) 76 GHz e) none above

$$\frac{m}{2h\sqrt{\epsilon\mu}} = \frac{1}{2(0.001)\sqrt{3.8}} = 76.9 \text{ GHz}$$

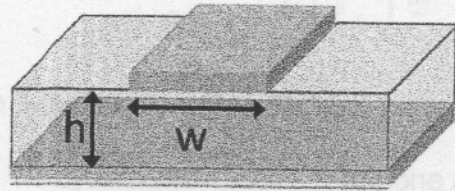
18. For the waveguide below where the H-field only contains components along the y axis, the mode is

- a) TE b) TM c) TEM d) y-mode e) none above



5 Points Each, Circle the best answer

$$w/h > 1$$



19. The effective relative permittivity of a microstrip line above of width $w=3$ mm on a 1 mm thick printed circuit board having $\mu=\mu_0$, $\epsilon=8\epsilon_0$, is $\epsilon_{eff}=?$

- a) 2.1 b) 3.9 c) 4.8 **d) 6.1** e) none above

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(\frac{1}{1 + 12h/w} \right) = \frac{9}{2} + \frac{7}{2\sqrt{1+4}} = 4.5 + 1.6 = 6.1$$

20. The impedance in ohms of a microstrip line above of width $w=3$ mm on a 1 mm thick printed circuit board having $\mu=\mu_0$, $\epsilon=8\epsilon_0$, is $Z_0=?$

- a) 28** b) 35 c) 73 d) 89 e) none above

$$Z_0 = \frac{377}{\sqrt{\epsilon_{eff}} (1.393 + w/h + 0.667 \ln(w/h + 1.4))} = \frac{377}{\sqrt{6.1} (1.393 + 3 + 0.99)} = 28$$

21. For the following propagation constants γ in microstrip using various materials, choose the value of γ that has the least loss.

- a) $2 + j5$ b) $2 + j9$ c) $4 + j5$ **d) $j7$**

22. The skin depth at 1 MHz for a conductor with $\sigma=10^3$ S/m, $\mu=\mu_0$, $\epsilon=4\epsilon_0$ is $\delta=?$

- a) 8 mm b) 11 mm **c) 16 mm** d) 24 mm e) none above

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{2}{2\pi \cdot 10^6 \cdot 1.257 \times 10^{-9} \cdot 10^3}} = \sqrt{\frac{1}{\pi \cdot 1.257}} = 16 \text{ mm}$$

23. The TE_{10} mode in rectangular waveguide with $h=4$ cm and $w=8$ cm has cutoff frequency $f_c=?$

- a) 1.9 GHz** b) 3.2 GHz c) 4.6 GHz d) 7.5 GHz e) none above

$$f_c = \frac{1.5 \times 10^8}{a} = \frac{0.15 \text{ GHz}}{0.08} = 1.88$$

24. For a 1 GHz plane wave in vacuum with normal incidence into a material with reflection coefficient $\Gamma = 0.5$, the ratio of reflected power P_r to incident power P_i is $P_r/P_i=?$

- a) -3 dB **b) -6 dB** c) -9 dB d) -12 dB e) none above

$$\Gamma = \frac{V_r}{V_{inc}} \quad \left| \frac{P_{ref}}{P_{inc}} \right| = \left| \frac{V_r^2}{V_i^2} \right| = |\Gamma|^2 = \frac{1}{4}$$

5 Points Each, Circle the best answer

$$10 \log_{10} (1441) = 31.5$$

25. The gain of an ideal ($\eta=1$) parabolic dish antenna with 1 meter diameter at 3.6 GHz is

- a) 21.1 dB **b) 31.5 dB** c) 39.5 dB d) 52.2 dB e) none above

$$\lambda = \frac{c}{f} = 0.083 \quad A_e = \pi r^2 = \pi \left(\frac{1}{2}\right)^2 = 0.79 \quad g = 4\pi A_e / \lambda^2 = 4\pi (0.79) / (0.083)^2 = 1441$$

26. The free-space propagation loss between two dipole antennas dBi at a distance of 6 km at 2 GHz is

- a) 99 dB **b) 110 dB** c) 133 dB d) 156 dB e) none above

$$\lambda = \frac{c}{f} = 0.15 \quad L = -10 \log_{10}(\lambda^2) + 10 \log_{10}((4\pi d)^2) - G_1 - G_2 = -16.5 + 97.5 - 2 - 2 = 110$$

27. The urban ($n=2.6$) propagation loss between two dipole antennas dBi at a distance of 6 km at 2 GHz is

- a) 99 dB b) 110 dB **c) 133 dB** d) 156 dB e) none above

$$L = 10 \log_{10}(\lambda^2) + 10 \log_{10}((4\pi d)^2 d^{2.6}) - G_1 - G_2 = -16.5 + 120.2 - 4 = 132.7$$

28. If the propagation loss between antennas is 100 dB, and transmitter power is 1 watt, and the receiver antenna impedance is 50 ohms, the received signal rms voltage is

- a) 320 nV b) 22 μ V **c) 71 μ V** d) 9.1 mV e) none above

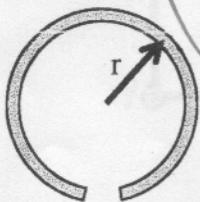
$$P_{rx} = 10^{-10} \text{ W} = \frac{V_{rms}^2}{50} \quad V_{rms} = \sqrt{50 \cdot 10^{-10}} = 70.7 \mu\text{V}$$

29. The Wheeler-Chu limit bandwidth of an electrically-small antenna with radius 5 mm at 1 GHz is

- a) 710 kHz **b) 1.1 MHz** c) 1.8 MHz d) 11 MHz e) none above

30. Using a ring inductance $L_R \approx 3.5 r \mu_0$, calculate the resonant frequency of the split ring resonator below, where the radius is $r=0.005$ m and the gap capacitance is $C_g=0.25$ pF.

- a) 1.6 GHz **b) 2.1 GHz** c) 2.9 GHz d) 8.3 GHz e) none above



$$L = 3.5 \cdot 0.005 \cdot 1257 \times 10^{-9} = 22 \text{ nH}$$

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f \quad f = \frac{1}{2\pi \sqrt{LC}} = 2.1 \text{ GHz}$$

5 Points Each, Circle the best answer

31. In a plane wave with propagation constant $\gamma = 2 + j\omega^3/6$, the group velocity is $v_g =$

- a) $2/\omega^2$ m/s b) $4/\omega^2$ m/s c) $2\omega^2$ m/s d) $j4\omega^2/3$ m/s e) none above

$$\beta = \frac{\omega^3}{6} \quad v_g = \frac{d\omega}{d\beta} = \frac{1}{d\beta/d\omega} = \frac{1}{3\omega^2/6} = \frac{2}{\omega^2}$$

32. A valid solution of the Helmholtz equation $\nabla^2 \mathbf{E} = -9\omega^2 \mathbf{E}$ is $\mathbf{E} =$

- a) $[1 \ 0 \ 0]^T e^{-j3\omega z}$ b) $[1 \ 0 \ 0]^T e^{-3\omega z}$ c) $[1 \ 0 \ 0]^T e^{-9\omega z}$ d) $[1 \ 0 \ 0]^T e^{j9\omega z}$ e) none above

$$\gamma^2 = -9\omega^2 \Rightarrow \gamma = \pm j3\omega$$

33. For a plane wave propagating in a right-handed metamaterial having $\sigma=0$, $\mu=\mu_0$, $\epsilon=0.25 \epsilon_0$, the phase velocity in terms of the speed of light ($c=3 \times 10^8$) is $v_p =$

- a) 0.25 c b) 0.5 c c) 0.71 c d) 2 c e) none above

$$\frac{c}{\sqrt{\epsilon_r}} = \frac{c}{\sqrt{0.25}} = 2c$$

34. For the boundary below between two media with $\sigma_1=0$, $\mu_1=\mu_0$, $\epsilon_1=\epsilon_0$ and $\sigma_2=0$, $\mu_2=\mu_0$, $\epsilon_2=4\epsilon_0$ and incident plane wave with $\theta_{inc} = 30$ degrees, the tangential E-field is zero at the boundary.

- a) True b) False

35. For the boundary below between two media with $\sigma_1=0$, $\mu_1=\mu_0$, $\epsilon_1=\epsilon_0$ and $\sigma_2=0$, $\mu_2=\mu_0$, $\epsilon_2=4\epsilon_0$ and an incident plane wave with $\theta_{inc} = 30$ degrees, the reflection angle in degrees is $\theta_{ref} =$

- a) 15 b) 30 c) 45 d) 60 e) none above

